IJournals: International Journal of Software & Hardware Research in Engineering
ISSN-2347-4890
Volume 4 Issue 1 January, 2016

Set Domination Maxsubdivision Number of Graphs

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ABSTRACT

Let G=(V,E) be a simple, undirected, finite nontrivial graph. A non empty set S⊆V of vertices in a graph G is called a dominating set if every vertex in V-S is adjacent to some vertex in S. The domination number γ(G) is the minimum cardinality of a dominating set of G.A dominating set S is a set dominating set of G if for every set T⊆V-S , there exists a non-empty set R⊆S such that the subgraph <RUT> is connected. The set domination number of G is the minimum cardinality of a set dominating set of G and it is denoted by γ_s (G).The set domination maxsubdivision number of G is the maximum number of edges that must be subdivided (where each edge in G can be subdivided atmost once) in order to increase the set domination number and is denoted by msdγ_s(G). In this paper, we establish the properties and exact values of the set domination maxsubdivision number for some families of graphs.

Keywords:

Dominating Set, Set domination number, MaxSet domination subdivision number.

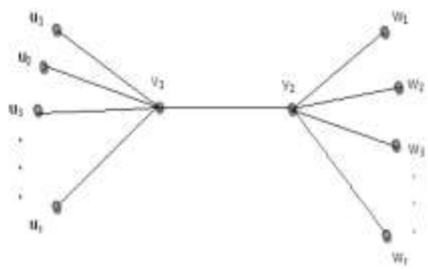
1.INTRODUCTION

Let G=(V,E) be a simple, undirected, finite nontrivial graph. A non empty set S⊆V of vertices in a graph G is called a dominating set if every vertex in V-S is adjacent to some vertex in S. The domination number γ(G) is the minimum cardinality of a dominating set of G.A dominating set S is a set dominating set of G if for every set T⊆V-S , there exists a non-empty set R⊆S such that the subgraph <RUT> is connected. The set domination number of G is the minimum cardinality of a set dominating set of G and it is denoted by γ_s (G).The set domination maxsubdivision number of G is the maximum number of edges that must be subdivided (where each edge in G can be subdivided atmost once) in order to increase the set domination number and is denoted by msdγ_s(G).

For notation and graph theory terminology, we in general follow [3]. Specially, a graph G is a finite nonempty set V(G) of objects called vertices (or) nodes together with a possibly empty set E(G) of 2-element subsets of V(G) called edges. The order of G is n(G)=|V(G)| and the size of G is m(G)= |E(G)|.The degree of of vertex vεV(G) in G is $d_G(v)= |N_G(v)|$. A vertex of degree one is c alled an end-vertex. The minimum and maximum degree among the vertices of G is denoted by δ(G) and Δ(G) respectively. A vertex of degree one is called an Pendant vertex. Any vertex is adjacent to a pendant vertex is called a support. A caterpillar is a tree for which the nodes that are not end nodes induce a path.

Example:1.1

Consider the following graph G:



Here the set dominating set $S=\{v_1,v_2\}$. Therefore $\gamma_s(G)=2$

There are three cases namely subdivision of v_1v_2 (or)u_iv_1 where $1\le i\le s$ (or) v_2w_j where $1\le j\le r$.

Case:1

Suppose subdivide the edge v_1v_2..

If $T=\{ u_iw_j\}$ where $1\le i\le s$ and $1\le j\le r$ there exists no $R\subseteq S$ such that $<R\cup T>$ is connected. Therefore set domination property is not satisfied. In this case msd $\gamma_s(G)=1$.

Case:2

Suppose subdivide u_iv_1 where $1\le i\le s$

In the subdivided graph u_i is not dominated by any vertex. Hence domination property is not satisfied. In this case msd $\gamma_s(G)=1$.

Case:3

Suppose subdivide v_2w_j where $1\le j\le r$.

In the subdivided graph w_j is not dominated by any vertex. Hence domination property is not satisfied. In this case msd $\gamma_s(G)=1$. Hence for the above possibilities cases, msd $\gamma_s(G)=1$.

The set dominating maxsubdivision number for several standard graphs are given in the following propositions.

Proposition:1.2

Let G be a complete bipartite graph with a partition (V_1,V_2) where $V_1=\{u_1,u_2,\ldots,u_m\}$ and $V_2=\{v_1,v_2,\ldots,v_n\}$ then msd γ_s (G)=2.

Proof:

Let G be a complete bipartite graph. Therefore the set dominating set γ_s (G) is $\{u_i,v_j\}$where $1\le i\le m$ and$1\le j\le n$. Therefore γ_s (G) =2. Subdivide the edge u_iv_j where $1\le i\le m$ and$1\le j\le n$ then the set dominating set is not affected. Therefore again Subdivide any edge in G then the set dominating set is affected. Hence domination property is not satisfied. Hence msd γ_s (G)=2.

Proposition:1.3

Let G be a path on n vertices say $v_1,v_2,\ldots,v_n$ where $n\ge 3$ where v_1 and v_n are pendant vertices and $v_2,v_3,\ldots,v_{n-1}$ are of degree 2 .Then sd γ_s (G)=1.

Proof:

In a path the set domination number γ_s (G) =n-2.Also if subdivide the edgev_sv_{s+1} where $2\le s\le n-1$

If $T=\{ v_1,v_n\}$ then there exists no $R\subseteq S$ such that $<R\cup T>$ is connected. Therefore set domination property is affected. Therefore msd $\gamma_s(G)=1$.

Proposition:1.4

Let G be a cycle with n vertices say $v_1,v_2,\ldots,v_n$ where n=3 and $n\ge 5$ .Then sd γ_s (G)=1.

Proof:

Case:1

Let n=3.Then $\gamma_s(G)=1$.Subdivide any edge v_iv_j where $i\ne j$ then the domination property is affected. Therefore msd $\gamma_s(G)=1$.

Case:2

Let $n\ge 5$.Subdivide any edge v_iv_j where $i\ne j$ then the domination property is affected. Therefore

msd $\gamma_s(G)=1$.

Remark:1.41

Let G be a cycle with n vertices say $v_1,v_2,\ldots,v_n$ where n=4 .Then msd γ_s (G)=2.

Proof:

Subdivide any edge in G then the domination property is not affected. Hence again subdivide any edge in G then the set domination number is increased. Therefore msd $\gamma_s(G)=2$.

Theorem:1.5

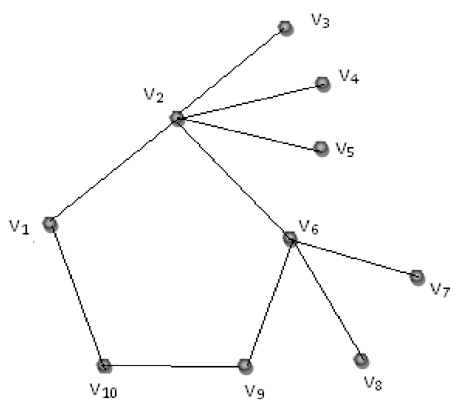
For any connected graph G, $1\leq$ msd $\gamma_s(G)\leq 3$.

Proof:

It appears that this theorem may be difficult to settle , either to show that it is true , or to find a counter example, since the theorem is a statement about the totality of all $\gamma_s(G)$-sets in a graph G and the effects that edge subdivisions must have on every $\gamma_s(G)$-set.

Example:1.51

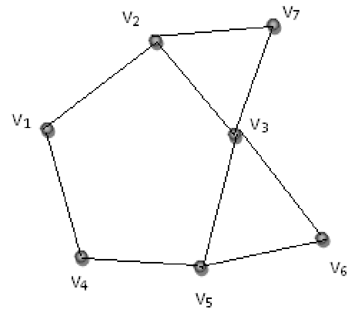
Consider G is a graph as shown in the following graph:



Here V={ $v_1,v_2,\ldots,v_{10}$} and the γ_s-set is { v_2,v_6,v_{10}}. Therefore $\gamma_s(G)=3$. But if subdivide any edge in G then the domination property (or) set domination property will be affected. Hence msd $\gamma_s(G)=1$.

Example:1.52

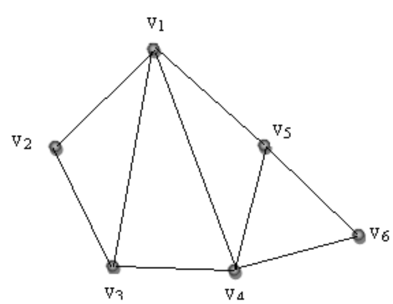
Consider G is a graph as shown in the following graph:



Here V={ $v_1,v_2,\ldots,v_7$} and the γ_s-set is { v_3,v_4,v_5}. Therefore $\gamma_s(G)=3$. But if subdivide the edges v_3v_6 and v_4v_5 then the set domination property is affected. Hence msd $\gamma_s(G)=2$.

Example:1.53

Consider G is a graph as shown in the following graph:



Here V={ $v_1,v_2,\ldots,v_6$} and the γ_s-set is { v_1,v_4}. Therefore $\gamma_s(G)=2$. If subdivide the edges v_1v_5 and v_3v_4 then the set domination number is not affected. Hence again subdivide v4v5 then the domination number is affected. Hence msd $\gamma_s(G)=3$.

Theorem:1.6

Let G be a caterpillar. Then msd $\gamma_s(G)=1$.

Proof:
Let the pendant vertices of the caterpillar be $w_1,w_2,\ldots,w_m$. Let V_1={ $w_1,w_2,\ldots,w_m$}. After deleting all the pendant vertices from G we get a

path P_n. Let it be $v_1,v_2,\ldots,v_n$. Here all the supports are to be in set dominating set.

Case:1

Subdivide the edge $v_s v_{s+1}$ where $2 \le s \le n-1$

If $T=\{w_1,v_n\}$ then there exists no $R \subseteq S$ such that $<R \cup T>$ is connected. Therefore set domination property is not satisfied. In this case msd $\gamma_s(G)=1$.

Case:2

Subdivide the edge $v_i v_j$ where i=1,n and j=2,n-1 then in the subdivided graph v_i is not dominated by any vertex. Therefore the domination property is affected.Hence msd $\gamma_s(G)=1$.

Case:3

Subdivide the edge $w_i v_j$ where $1 \le i \le m$ and $1 \le j \le n$ then in the subdivided graph w_i is not dominated by any vertex. Therefore the domination property isaffected. Hence msd $\gamma_s(G)=1$.

Theorem:1.7

Let G be a connected graph. If deg v=n-1 then msd $\gamma_s(G)=1$.

Proof:

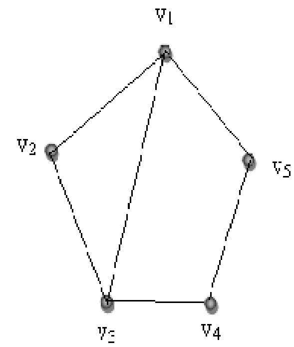
Let G be a connected graph and let $v \varepsilon G$. Let degv=n-1. Then v is adjacent to all other vertices of G. If subdivide any one edge in G then the domination property is affected. Hence msd $\gamma_s(G)=1$.

Remark:1.71

The converse of the above theorem is not true.

Example:1.72

Consider G as follows :



Here $\gamma_s(G)=1$.If Subdivide the edge v_4v_5 then msd $\gamma_s(G)=1$.

Definition:1.8

Let G be a connected graph. If G is said to be ladder then u_1,u_r, v_1,v_s be the pendant vertices of G and all other vertices of degree 3 and u_i is adjacent to v_i for $2 \le i \le s-1$ and u_i is adjacent to u_{i+1} for $1 \le i \le s-1$.

Theorem:1.9

For a ladder , msd $\gamma_s(G)=2$ if the following conditions (i) and (ii) hold.

(i) subdivide $v_i v_j$ where $v_i \varepsilon S$ and $v_j \notin S$

(ii) Subdivide any edge in G.

Proof:

Let the pendant vertices of the ladder be $w_1,w_2,\ldots,w_m$ and $v_1,v_2,\ldots,v_n$ be the vertices of G with deg vi=3 where $1 \le i \le n$.

Case:1

Subdivide $w_i v_j$ $1 \le i \le m$, $1 \le j \le n$.

The set dominating set contains p/2 vertices that does not contain any pendant vertices. Hence $\gamma_s(G)=p/2$.

If Subdivide $w_i v_j$ $1 \le i \le m$, $1 \le j \le n$ then the domination property is affected. Hence msd $\gamma_s(G)=1$.which is a contradiction to maxsubdivision property.

Case:2

Subdivide v_iv_j where $v_i \varepsilon S$ and $v_j \varepsilon S$. Then for T={ w_1,w_4} $\subseteq$V-S there exists no R$\subseteq$S such that <R$\cup$T> is connected. Therefore set domination property is affected. msd γ_s(G)=1.which is a contradiction to maxsubdivision property.

Case:3

If subdivide v_iv_j where $v_i \varepsilon S$ and $v_j \notin S$ then the set domination property is not affected. Since the new subdividing vertex is dominated by v_i's where $v_i \varepsilon S$.Also, $v_j \notin S$is dominated by any one of v_i's where $v_i \varepsilon S$. Hence again subdivide any edge in G then the set dominating property (or) domination property is affected. Hence msd γ_s(G)=2.

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