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A Note on Non-Split Set Domination

[1]P. Sumathi, [2]T.Brindha
[1]Head & Associate Professor, Department of Mathematics, C.K.N College for Men, Anna Nagar, Chennai – 102,Tamilnadu,India.
[2]Assistant Professor ,Department of Mathematics, E.M.G. Yadava Women's College, Madurai – 625014,Tamilnadu, India.
sumathipaul@yahoo.co.in, thiruppathibrindha@yahoo.com

ABSTRACT

Let G=(V,E) be a simple, undirected, finite nontrivial graph. A non empty set S⊆V of vertices in a graph G is called a dominating set if every vertex in V-S is adjacent to some vertex in S. The domination number γ(G) of G is the minimum cardinality of a dominating set of G.A dominating set S is called a non split set dominating set if there exists a non empty set R⊆ S such that <R∪T> is connected for every set T⊆V-S and the induced subgraph <V-S> is connected. The minimum cardinality of a nonsplit set dominating set is called the non split set domination number of G and is denoted by

γ_{nss} (G). In this paper, bounds for γ_{nss} (G) and exact values for some particular classes of graphs are found.

Keywords:
Dominating Number, Non Split domination number.

1.INTRODUCTION

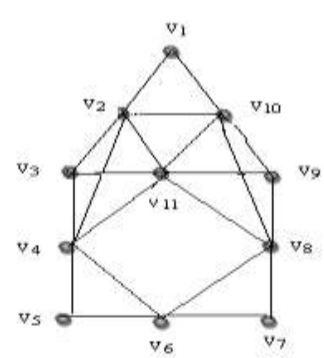
Let G=(V,E) be a simple, undirected, finite nontrivial graph with vertex set V and edge set E. The maximum, minimum degree among the vertices of G is denoted by Δ(G),δ(G) respectively. If deg v=0 then v is called an isolated vertex of G. If deg v=1 then v is called a pendant-vertex of G. And K_n,C_n,P_n and $K_{1,n-1}$ denote the complete graph, the cycle, the Path and the Star on n vertices respectively.

A non empty set S⊆V of vertices in a graph G is called a dominating set if every vertex in V-S is adjacent to some vertex in S. The domination number γ(G) of G is the minimum cardinality of a dominating set of G.A dominating set S of a graph G=(V,E) is a non split set dominating set if there exists a non empty set R⊆ S such that <R∪T> is connected for every set T⊆V-S and the induced subgraph <V-S> is connected. The non split domination number γ_{ns} (G) of a graph G is the minimum cardinality of a nonsplit dominating set. A set S is said to be a γ_{ns} –set if S is a minimum non split dominating set.

Definition:1.1

A dominating set S is said to be nonsplit set dominating set if there exists a nonempty set R⊆ S such that <R∪T> is connected for every set T⊆V-S and the induced subgraph <V-S> is connected. The minimum cardinality of a nonsplit dominating set is called the non split domination number of G and is denoted by γ_{nss} -set if S is a minimum non split dominating set.

Example: 1.2
Consider the following graph G:



Here the Set dominating set $S=\{v_1,v_6,v_{11}\}$.And $V-S=\{v_1,v_2,v_3,v_4,v_5,v_7,v_8,v_9\}$ Here for every $T\subseteq V-S$ there exists a nonempty set $R\subseteq S$ such that $\langle R\cup T\rangle$ is connected and also the induced subgraph $\langle V-S\rangle$ is connected. $\gamma_{ss}(G)=3$.

2. CHARACTERIZATION OF NON-SPLIT SET DOMINATING SET:

Observation: 2.1

1. For any connected graph G, $\gamma(G)\leq \gamma_{nss}(G)$.

2. For any connected spanning subgraph H of G, $\gamma_{nss}(G)\leq \gamma_{nss}(H)$.

Example: $\gamma_{nss}(K_4)=1$, $\gamma_{nss}(C_4)=2$.

Proposition: 2.2

Pendant vertices are members of every nss-set.

Proof:

Let v be any vertex in G such that deg v=1 and let S be a nss-set.If $v\varepsilon V-S$ then a vertex adjacent to v must be in S and hence $\langle V-S\rangle$ is disconnected ,which is a contradiction.

Proposition: 2.3

$\gamma_{nss}(G)\geq e$ where e is the number of pendant vertices.

Proof:

Since every pendant vertex is a member of each non split set dominating set.

3. BOUNDS OF NON-SPLIT SET DOMINATION NUMBER:

Observation: 3.1

For any connected graph G with n>3, $\gamma_{nss}(G)\leq n-2$.

This bound is sharp for C_n and P_n.

Theorem:3.2

Let G be a complete graph with n vertices say $v_1,v_2,\ldots,v_n$ $(n\geq 3)$ then $\gamma_{nss}(G)=1$.

Proof:

The set dominating set of a complete graph G is $\{v_1\}$ with cardinality 1. Here, $V-S=\{v_2,v_3,\ldots,v_n\}$ .Also the induced subgraph $\langle V-S\rangle$ is connected. Hence $\gamma_{nss}(G)=1$.

Theorem: 3.3

Let G be a connected graph with no pendant vertices with deg v= n-1 where $v\varepsilon G$ if and only if $\gamma_{nss}(G)=1$.

Proof:

Let deg v=n-1. Therefore v is adjacent to every vertex in G. Here S={v}. Also $\langle V-S\rangle$ is connected. Therefore $\gamma_{nss}(G)=1$.Conversely, let $\gamma_{nss}(G)=1$. Therefore $v\varepsilon S$ is adjacent to every other vertex of G. Hence deg v=n-1.

Theorem:3.4

Let G be a complete bipartite graph $K_{m,n}$ with $m,n\geq 2$. Then $\gamma_{nss}(G)=2$.

Proof:

Let $V_1=\{u_1,u_2,\ldots,u_m\}$ and $V_2=\{v_1,v_2,\ldots,v_n\}$ be two partitions of G . And $V_1\cup V_2=V$ and $V_1\cap V_2=\phi$. The set dominating set is $\{u_1,v_1\}$.Also $V-S=\{u_2,u_3,\ldots,u_m,v_2,v_3,\ldots,v_n\}$. Here the induced subgraph $\langle V-S\rangle$ is connected. Hence $\gamma_{nss}(G)=2$.

Theorem: 3.5

Let G be a connected graph with pendant vertices. Then $\gamma_{nss}(G)=n-1$ if and only if G is a star for all $n\geq 2$.

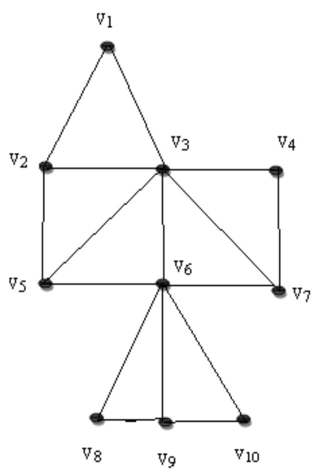
Proof:

Let $\gamma_{nss}(G)=n-1$. Therefore V-S has only one element . Therefore the induced subgraph $\langle V-S\rangle$ is trivial and is connected. Therefore S has n-1 vertices and they should be pendant vertices and also they are adjacent to a common vertex u. hence G must be a star. Conversely , let G be a star with star center u and let $u_1,u_2,\ldots,u_{n-1}$ be the pendant vertices of G. The set dominating set is $\{u,u_1,u_2,\ldots,u_{n-2}\}$ or $\{u_1,u_2,\ldots,u_{n-1}\}$ with cardinality 1+n-2 (i.e) n-1. Also V-S=$\{u_n\}$ or $\{u\}$. Therefore the induced subgraph <V-S> is trivial and every trivial graph is connected. Hence <V-S> is connected. Therefore γ_{nss} (G)= n-1 for all n≥2.

4. RELATION BETWEEN NON-SPLIT SET DOMINATION NUMBER AND OTHER PARAMETER:

Observation: 4.1

For any connected graph G with no pendant vertex, $p-\Delta(G)\leq \gamma(G) +\gamma_{nss}(G)\leq p+\delta(G)$.



Here the Dominating set is $\{v_3,v_9\}$. Therefore $\gamma_{ss}(G)=2$. The Set dominating set S=$\{v_3,v_6,v_9\}$And V-S=$\{v_1,v_2,v_4,v_5,v_6,v_7,v_8,v_{10}\}$.Here for every T⊆V-S there exists a nonempty set R⊆ S such that <R∪T> is connected and also the induced subgraph <V-S> is connected. Hence $\gamma_{ss}(G)=3$.
Here p=10, $\Delta(G)=6,\delta(G)=2$.
Therefore $p-\Delta(G)\leq \gamma(G) +\gamma_{nss}(G)\leq p+\delta(G)$.

Proposition:4.2

Let G be a complete graph with n vertices then $\gamma_{nss}(G)+\Delta(G)=n$.

Proof:

For any complete graph G, $\Delta(G)=n-1$ and by the theorem 3.2 , $\gamma_{nss}(G)=1$. Hence proved.

Proposition: 4.3

$\gamma_{nss}(G)+e=n$ where e is a pendant vertex if and only if G is a star.

Proof:

By the theorem 3.5, $\gamma_{nss}(G)=n-1$ then G is a star. Therefore $\gamma_{nss}(G)+e=n$. Conversely , let $\gamma_{nss}(G)+e=n$. Hence $\gamma_{nss}(G)=n-1$. Therefore G must be a star.

Theorem: 4.4

For any tree T, which is not a star then $\gamma_{nss}(T)=n-2$ for all n>4.

Proof:

Let $V_1=\{u_1,u_2,\ldots,u_l\}$ be the set of all pendant vertices of T and let $V_2=\{s_1,s_2,\ldots,s_k\}$ be the set of all supports of T and let $V_3=\{v_1,v_2,\ldots,v_s\}$ be the set of all vertices of degree greater than one which are not a support. Let $V=V_1\cup V_2\cup V_3=\{v_1,v_2,\ldots,v_n\}$ be the vertex set of T. Then the set dominating set is $(V_2\cup V_3)\setminus V_1$. But the induced subgraph <V\S> is disconnected. Therefore the non-split set dominating set is V-$\{s_i\}$-$\{v_j\}$ where 1≤i≤k and 1≤j≤s. The cardinality of the non split set dominating set is n-2. Therefore V-S has two vertices. Also the induced subgraph <V-S> is connected. Hence $\gamma_{nss}(G)=n-2$ for all n>4.

Theorem: 4.5

Let G be a ladder then $\gamma_{nss}(G)=n-2$.

Proof:

Let $V_1=\{u_1,u_2,\ldots,u_s\}$ and $V_2=\{v_1,v_2,\ldots,v_l\}$ then each ui is adjacent to u_{i+1}, 1≤i≤s-1 and each v_j is adjacent to v_{j+1}, 1≤j≤l-1. Also u_i is adjacent to v_j for all i=j and u_1,u_s,v_1,v_l are pendant vertices. The set dominating set S is V-$\{u_i,v_j\}$ (or) V-$\{u_i,u_{i+1}\}$ (or) V-$\{v_j,v_{j+1}\}$ where 2≤i≤s-1 and 2≤j≤l-1. Hence the induced subgraph is connected with two vertices. Hence γ_{nss} (G)=n-2.

Theorem: 4.6

For any connected graph G, γ_{nss} (G)=e where e is the number of pendant vertices if and only if G is a star (or) G is a bistar

Proof:

Let γ_{nss} (G)=e. Therefore all pendant vertices are members of γ_{nss} (G). Also there exists $R\subseteq S$ such that <R∪T> is connected for every set $T\subseteq V-S$ and the induced subgraph <V-S> is connected. Hence <V-S> is trivial graph or it is connected with two vertices. Hence G must be a star (or) G is a bistar. Conversely , the non-split set dominating set has all pendant vertices to satisfy the non-split set dominating set property. Therefore V-S has a single vertex or two adjacent vertices. Therefore the induced subgraph <V-S> is connected. Hence γ_{nss} (G)=e.

Theorem: 4.7

Any cycle with p vertices has two pendant vertices with two supports then γ_{nss} (G) =p-e=p-s.

Proof:

All pendant vertices and all supports are in non split set dominating set. And γ_{nss} (G)=p-2 for any cycle. Hence γ_{nss} (G)=p-2=p-e=p-s. Hence proved.

Theorem:4.8

A non split set dominating set S is minimal if and only if each uεS satisfies one of the following four conditions.

(i) $N(u)\cap S=\phi$

(ii) There exists an independent set $S_0\subseteq V-S$ such that $\bigcap_{s\in S_0}(N(s))\cap S$ ={u}

(iii) there exists an independent set $S_0\subseteq V-S$ with $N(u)\cap S=\phi$ and S0 is not connected in N(v) for any

vεN(u) ∩S in N(v) for any vεN(u) ∩S

(iv) $N(u)\cap(V-S)=\phi$.

Proof:

Let S be a minimal non split set dominating set. Then either S-{u} is not a set dominating set (or) S-{u} is a set dominating set but not a non split set dominating set.

If S-{u} is not a set dominating set, then u satisfies one of the three conditions (i) to (iii).

If S-{u} is a set dominating set but not a nonsplit set dominating set , then <V-S∪{u}> is not connected. But <V-S> is connected. Hence $N(u)\cap(V-S)=\phi$.

Conversely, suppose each uεS satisfies one of the above four conditions. Let if possible S be not minimal. Then there exists uεS such that S-{u} is a nonsplit set dominating set. Then this u satisfy none of the above four conditions. Which is a contradiction. Hence Proved.

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REFERENCES

1. J.Cyman, The outer –connected domination number of a graph, Australian J.Comb.38(2007),35-46.

2. T.W.Haynes,S.T.Hedetniemi and P.J.Slater, Fundamentals of domination graphs, Marcel Dekker, Newyork,(1998).

3. Hongxing Jiang, Erfang Shan, outer-connected domination in graphs, Utilities Mathematics 81 (2010).

4. V.R.Kulli and B. Janakiram , The non split domination number of a graph, India, J. Pure Appl. Math; 31 (2000) 545-550.