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Truncated Spline For Estimating the Curve of Nonparametric Regression Bi-Responses For Prediction The Inflation and Economic Growth in Malang, Indonesia, Year 2001-2015

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ABSTRACT

This study aims to obtain nonparametric regression curve estimation based truncated spline, especially for models that involve more than one response, in this case use two responses or bi-responses. The data used in this study are the Minimum Living Needs (X1) and the Regional Minimum Wage (X2) as a predictor variable, and Inflation (Y1) and Economic Growth (Y2) in Malang, Year 2001-2015. Data sourced from reports the Central Bureau of Statistics (2016). Based on the results and discussion, it can be concluded that the effect of Minimum Living Needs (X1) and the Regional Minimum Wage (X2) on Inflation (Y1) and Economic Growth (X2) can be formed through the spline Truncated in Nonparametric Regression Bi-responses.

Keywords: Truncated Spline, Inflation, Economic Growth, Malang

1. INTRODUCTION

This study aims to obtain nonparametric regression curve estimation based truncated spline, especially for models that involve more than one response, in this case use two responses or bi-responses. Regression analysis is a statistical analysis used to determine the effect of predictor variables with the response variable (Hardle [1]). In the regression analysis to estimate the regression curve, there are two approaches, namely parametric and nonparametric regression. Called the parametric regression approach if the curve shape is known for example linear regression, quadratic, cubic, p^{th}-polynomial. In nonparametric regression approach regression curve shape is unknown.

Parametric regression is a regression model that assumes the pattern of the relationship between the response and predictors can be described in a specific function such as the pattern of straight lines, polynomial, or exponential, or other. In the application, to obtain the proper function is very difficult and often find symptoms that indicate that the data obtained is not or has not shown a pattern of relationships that are easy to describe. Wu and Zhang [2] states that a parametric regression model must meet the assumption of a linear relationship between the response form with the predictor. If the assumptions are not met linearity and shape of non-linearity is not known, then one alternative that can be used is a nonparametric regression model.

Nonparametric regression is a regression approach that is appropriate for the pattern of relationship between predictors with a response that is not known shape, or there is no information about the past relationship patterns (Eubank, [3]; Budiantara, [4]). In nonparametric regression approach, form relationships pattern model estimation is based on an existing data pattern. The pattern of the relationship between the response predictors unknown can be estimated using the approach function Truncated spline (Craven and Wahba [5]; Wahba, [6]; Budiantara, Subanar & Soejoeti [7]), polynomial Local (Fan and Gijbels [8]), Kernel (Hardle [9]), Wavelets (Antoniadis, Gregoire & McKeague, [10]), and Fourier series (Maliavin & Mancino, [11]). Spline truncated approach has high flexibility and is capable to handling the data relationship behavior patterns changing in subgroups specified interval (Eubank [3]). It has also been shown by Liang [12], which compares the truncated smoothing spline function with numerical kernel, and Aydin [13], which compares the truncated smoothing spline function with a kernel which is numerically better truncated spline function.

Regression models can be distinguished by the number of responses are involved, namely the regression model with multi-responses or single response. Single response model, as it has developed many previous researchers, consists of a single response that is affected by one or more predictors. On the other hand, the model multi-responses consists of several models with the assumption that there is a correlation or dependence between responses. Some researchers have studied the nonparametric regression multi-responses for cross-section data including Wang, Guo & Brown [14], Matias [15], and Lestari, Budiantara, Sunaryo, & Mashuri [16] approach to smoothing truncated spline, and Chamidah, Budiantara, Sunaryo, and Zain [17] with local polynomial approach. Basically, modeling multi-responses goal is to get a better model than a single response modeling, considering this model not only consider the effect of a predictor of response, but also the relationship between the response. Representation of the relationship between the response is usually expressed in terms of the variance covariance matrix, which is used as a weighting in the model parameter estimation.

Nonparametric regression approach that is frequently used by the researchers is truncated spline. Truncated spline is a polynomial pieces which have segmented and continuous nature. Truncated spline is an estimator obtained by minimizing the Least Square (LS), namely optimization criteria that combines the goodness of fit and the smooth-curve. However, the completion of these optimization mathematically relatively difficult (Budiantara [4]). According to (Crainiceanu, C.M., Ruppert, D., & Wand [18]) the use of a truncated approach spline with the truncated polynomial basis optimization solution using least square (LS) may be a better choice. Truncated polynomial spline is the sum of the polynomial function with a function.

2. MODEL PROPERTIES

Not all data pairs of predictors and responses known patterns of relationship clearly. If the parametric model to be used as a model specific data patterns, then the conclusion would be invalid. Nonparametric regression is a regression approach methods appropriate for the data pattern of the unknown form of regression curves or there is no information about the previous data patterns (Eubank [3]). Nonparametric regression model with m and k predictors of response in cross-section data involving n observations as follows (Fernandes [19]):

$$y_{ki} = \sum_{\ell=1}^{m} f_{k\ell}(x_{\ell i}) + \varepsilon_{ki} \;;i=1,2,..,n \qquad (2.1)$$

The curve simply assumed to be smooth in the sense of being in a particular function space. Nonparametric regression approach has high flexibility, since the data is expected to find the shape of the regression curve estimation without being influenced by the subjectivity of the researcher (Eubank [3]). Several approaches are used to estimate the nonparametric regression function are kernel, truncated spline, Fourier series (Budiantara [4]). Nonparametric regression approach that is often used is truncated spline.

Truncated of a polynomial spline (piecewise polynomial), which is a polynomial that has properties of the continuous segmented. According Budiantara [7], a truncated spline regression model in which the model to adapt to the characteristics of the data. Truncated spline has an advantage in overcoming the pattern data showing a sharp rise or fall with the help of number and point of knots, as well as the resulting curve is relatively smooth.

One estimator of spline is truncated by K knots. Truncated spline truncated has advantages when compared to other estimator. Excellence truncated spline that can be used on the data to change behavior at certain intervals. In the truncated spline used power bases with K knots, the function is defined as follows:

$$(x_{\ell i} - k_{kj})_+^p = \begin{cases} (x_{\ell i} - k_{k\ell j})^p & ;x_{\ell i} \ge k_{k\ell j} \\ 0 & ;x_{\ell i} < k_{k\ell j} \end{cases} \qquad (2.2)$$

p indicates the degree of the polynomial truncated power bases, p = 1, 2, and 3 is generally a function of the degree of the polynomial linear, quadratic, and cubic (Fernandes [19]). The following general form with a truncated spline-p-order polynomial to the K point knots are as follows:

$$f_{k\ell}(x_{\ell i}) = \alpha_{k0} + \sum_{h=1}^{p} \alpha_{k\ell h} x_{\ell i}^{h} + \sum_{j=1}^{K} \beta_{k\ell j}(x_{\ell i} - k_{k\ell j})_+^p \qquad (2.3)$$

For example, a nonparametric regression model with spline truncated approach by the two number of responses or bi-responses and two number of predictors from n observations using equation (2.1) is as follows:

$$\begin{aligned} y_{1i} &= f_{11}(x_{1i}) + f_{12}(x_{2i}) + \varepsilon_{1i} \\ y_{2i} &= f_{21}(x_{1i}) + f_{22}(x_{2i}) + \varepsilon_{2i} \end{aligned} \qquad (2.4)$$

Equation (2.4) can be translated into the form of a matrix with i^{th} observation in Equation (2.5). Curve of spline nonparametric regression model truncated to two the number of responses, the two number of predictors in equation (2.4) where j = 1, 2, .. K and h = 1, 2, .. p, can be written in Equation (2.6).

$$\begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1n} \\ \\ y_{21} \\ y_{22} \\ \vdots \\ y_{2n} \end{bmatrix} = \begin{bmatrix} f_{11}(x_{11}) + f_{12}(x_{21}) \\ f_{11}(x_{12}) + f_{12}(x_{22}) \\ \vdots \\ f_{11}(x_{1n}) + f_{12}(x_{2n}) \\ \\ f_{21}(x_{11}) + f_{22}(x_{21}) \\ f_{21}(x_{12}) + f_{22}(x_{22}) \\ \vdots \\ f_{21}(x_{1n}) + f_{22}(x_{2n}) \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1n} \\ \\ \varepsilon_{21} \\ \varepsilon_{22} \\ \vdots \\ \varepsilon_{2n} \end{bmatrix} \quad (2.5)$$

$$y_{1i} = \alpha_{10} + \sum_{h=1}^{p} \alpha_{11h} x_{1i}^{h} + \sum_{j=1}^{K} \beta_{11j} (x_{1i} - k_{11j})_{+}^{p} + \sum_{h=1}^{p} \alpha_{12h} x_{2i}^{h} + \sum_{j=1}^{K} \beta_{12j} (x_{2i} - k_{12j})_{+}^{p} + \varepsilon_{1i} \quad (2.6)$$

$$y_{2i} = \alpha_{20} + \sum_{h=1}^{p} \alpha_{21h} x_{1i}^{h} + \sum_{j=1}^{K} \beta_{21j} (x_{1i} - k_{21j})_{+}^{p} + \sum_{h=1}^{p} \alpha_{22h} x_{2i}^{h} + \sum_{j=1}^{K} \beta_{22j} (x_{2i} - k_{22j})_{+}^{p} + \varepsilon_{2i}$$

$$\mathbf{x} = \begin{pmatrix} \mathbf{A}_{n\times(1+2p+2K)} & \mathbf{O}_{n\times(1+2p+2K)} \\ \mathbf{O}_{n\times(1+2p+2K)} & \mathbf{B}_{n\times(1+2p+2K)} \end{pmatrix}_{2n\times 2(1+2p+2K)}$$

$$\mathbf{A} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{11}^{p} & (x_{11}-k_{111})_{+}^{p} & \cdots & (x_{11}-k_{11K})_{+}^{p} & x_{21} & \cdots & x_{21}^{p} & (x_{21}-k_{121})_{+}^{p} & \cdots & (x_{21}-k_{12K})_{+}^{p} \\ 1 & x_{12} & \cdots & x_{12}^{p} & (x_{12}-k_{111})_{+}^{p} & \cdots & (x_{12}-k_{11K})_{+}^{p} & x_{22} & \cdots & x_{22}^{p} & (x_{22}-k_{121})_{+}^{p} & \cdots & (x_{22}-k_{12K})_{+}^{p} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1n} & \cdots & x_{1n}^{p} & (x_{1n}-k_{111})_{+}^{p} & \cdots & (x_{1n}-k_{11K})_{+}^{p} & x_{2n} & \cdots & x_{2n}^{p} & (x_{2n}-k_{121})_{+}^{p} & \cdots & (x_{2n}-k_{12K})_{+}^{p} \end{pmatrix}_{n\times(1+2p+2K)}$$

$$\mathbf{B} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{11}^{p} & (x_{11}-k_{211})_{+}^{p} & \cdots & (x_{11}-k_{21K})_{+}^{p} & x_{21} & \cdots & x_{21}^{p} & (x_{21}-k_{221})_{+}^{p} & \cdots & (x_{21}-k_{22K})_{+}^{p} \\ 1 & x_{12} & \cdots & x_{12}^{p} & (x_{12}-k_{211})_{+}^{p} & \cdots & (x_{12}-k_{21K})_{+}^{p} & x_{22} & \cdots & x_{22}^{p} & (x_{22}-k_{221})_{+}^{p} & \cdots & (x_{22}-k_{22K})_{+}^{p} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1n} & \cdots & x_{1n}^{p} & (x_{1n}-k_{211})_{+}^{p} & \cdots & (x_{1n}-k_{21K})_{+}^{p} & x_{2n} & \cdots & x_{2n}^{p} & (x_{2n}-k_{221})_{+}^{p} & \cdots & (x_{2n}-k_{22K})_{+}^{p} \end{pmatrix}_{n\times(1+2p+2K)}$$

Equation (2.6) can be denoted in the matrix and vector as follows:

$$\underset{\sim}{y} = \mathbf{x}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon} \quad (2.7)$$

The matrix **O** is that all elements of the matrix is zero (0) where n is the number of observations that will be examined with the matrix **A** and **B**.

If there is a correlation between residual in the first response to the residual in second response, then the calculation of correlation between residual for each response can be seen in equation (2.8). This value is assumed to be homogeneous but residual variance values in each response is not the same, then the variance-covariance matrix can be seen in equation (2.9) (Fernandes, [19]).

$$\rho = \frac{\operatorname{cov}(y_1, y_2)}{\sqrt{\operatorname{var}(y_1)\operatorname{var}(y_2)}} \quad (2.8)$$

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1^2 & 0 & \ldots & 0 & \sigma_{(1,1)} & 0 & \ldots & 0 \\ 0 & \sigma_1^2 & \ldots & 0 & 0 & \sigma_{(2,2)} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_1^2 & 0 & 0 & \ldots & \sigma_{(n,n)} \\ \sigma_{(1,1)} & 0 & \ldots & 0 & \sigma_2^2 & 0 & \ldots & 0 \\ 0 & \sigma_{(2,2)} & \ldots & 0 & 0 & \sigma_2^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_{(n,n)} & 0 & 0 & \ldots & \sigma_2^2 \end{pmatrix}_{(2n\times 2n)} \quad (2.9)$$

To obtain a regression curve estimator in equation (2.7), use the optimization by Weight Least Square (WLS), by completing the following equation

$$\min_{\underset{\sim}{\beta}\in R^{2(p+K+2)}} \left\{ \underset{\sim}{\varepsilon}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underset{\sim}{\varepsilon} \right\} = \min_{\underset{\sim}{\beta}\in R^{2(p+K+2)}} \left\{ (\underset{\sim}{y} - \mathbf{x}\underset{\sim}{\beta})^{\mathrm{T}} \mathbf{\Sigma}^{-1} (\underset{\sim}{y} - \mathbf{x}\underset{\sim}{\beta}) \right\} \quad (2.10)$$

To complete the optimization in equation (2.10), by partial derivatives, defined as follow

$$\begin{aligned} Q(\underset{\sim}{\beta}) &= (\underset{\sim}{y} - \mathbf{x}\underset{\sim}{\beta})^{\mathrm{T}} \mathbf{\Sigma}^{-1} (\underset{\sim}{y} - \mathbf{x}\underset{\sim}{\beta}) = (\underset{\sim}{y}^{\mathrm{T}} - \underset{\sim}{\beta}^{\mathrm{T}} \mathbf{x}^{\mathrm{T}}) \mathbf{\Sigma}^{-1} (\underset{\sim}{y} - \mathbf{x}\underset{\sim}{\beta}) \\ &= \underset{\sim}{y}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underset{\sim}{y} - \underset{\sim}{\beta}^{\mathrm{T}} \mathbf{x}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underset{\sim}{y} - \underset{\sim}{y}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \mathbf{x}\underset{\sim}{\beta} + \underset{\sim}{\beta}^{\mathrm{T}} \mathbf{x}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \mathbf{x}\underset{\sim}{\beta} \\ &= \underset{\sim}{y}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underset{\sim}{y} - 2\underset{\sim}{\beta}^{\mathrm{T}} \mathbf{x}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \underset{\sim}{y} + \underset{\sim}{\beta}^{\mathrm{T}} \mathbf{x}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \mathbf{x}\underset{\sim}{\beta} \end{aligned} \quad (2.11)$$

The next step is to determine the equation (2.11) on the parameters and equated to zero. Thus, parameter estimation can be written as follows

$$\frac{\partial Q(\tilde{\beta})}{\partial \tilde{\beta}} = -2\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\tilde{y} + 2\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{x}\tilde{\beta} \qquad (2.12)$$

$$\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\tilde{y} = \mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{x}\hat{\tilde{\beta}} \qquad (2.13)$$

$$\hat{\tilde{\beta}} = (\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{x})^{-1}\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\tilde{y} \qquad (2.14)$$

Equation (2.14) in a form truncated spline function estimation in nonparametric regression bi-responses as follows

$$\hat{\tilde{f}} = (\hat{f}_1, \hat{f}_2)^{\mathrm{T}}$$

$$\hat{\tilde{f}} = \mathbf{x}(\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{x})^{-1}\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\tilde{y}$$

$$\hat{\tilde{f}} = \mathbf{H}(\tilde{\mathbf{K}})\tilde{y} \qquad (2.15)$$

The matrix $\mathbf{H}(\tilde{\mathbf{K}})$ is a function of the point knots. Knot is a point in the truncated spline function, so that the curve formed segmented at that point. Selection of knots very important to choose the degree of polynomial, it is because the determinant of the accuracy of probe models. Choosing the optimal number and point of knots that produce value Generalized Cross Validation (GCV) minimum. Selection of the optimal knots by GCV method is defined as follows:

$$GCV(\tilde{K}) = \min\left[\frac{MSE(\tilde{K})}{\left[n^{-1}tr(\mathbf{I}-\mathbf{H}(\tilde{\mathbf{K}}))\right]^2}\right] \qquad (2.16)$$

$$MSE(\tilde{K}) = n^{-1}(\tilde{y}-\mathbf{x}\tilde{\beta})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\tilde{y}-\mathbf{x}\tilde{\beta}) \qquad (2.17)$$

$$\mathbf{H}(\tilde{\mathbf{K}}) = \mathbf{x}(\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{x})^{-1}\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1} \qquad (2.18)$$

3. MODEL IMPLEMENTATION

The data used in this study are the Minimum Living Needs (X1) and the Regional Minimum Wage (X2) as a predictor variable, and Inflation (Y1) and Economic Growth (Y2), in Malang, 2001-2015. Data sourced from reports the Central Bureau of Statistics (2016). The pattern of the relationship between the response variable by the predictor variable can be seen in Figure 1 and Figure 2.

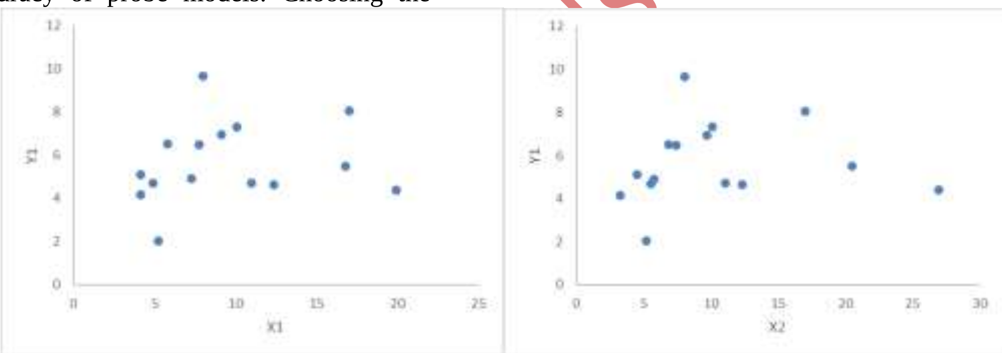

Figure 1 Scatterplot between Minimum Living Needs (X1) and the Regional Minimum Wage (X2) on Inflation (Y1)

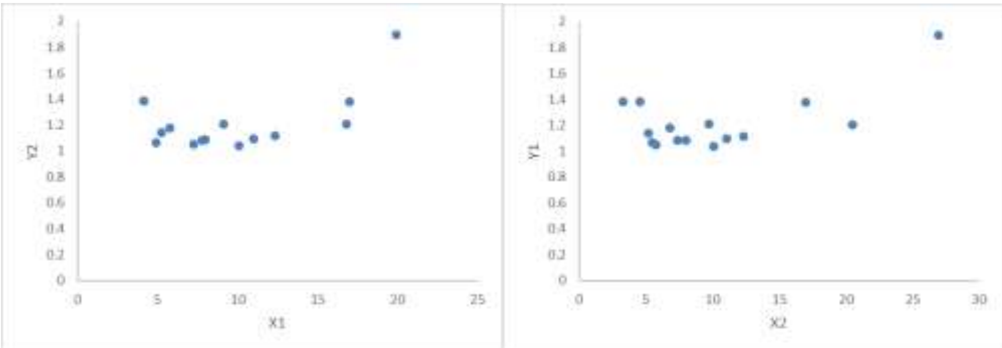

Figure 2 Scatterplot between Minimum Living Needs (X1) and the Regional Minimum Wage (X2) on Economic Growth (Y2)

Figure 1 and Figure 2 shows the pattern of relationships formed between Minimum Living Needs (X1) and the Regional Minimum Wage (X2) on Inflation (Y1) and Economic Growth (X2) do not show clearly a particular pattern. So that the approach can be used is nonparametric regression. In addition to using a scatter plot to see whether the data can use nonparametric regression or not can also use the linearity test. Linearity test aims to determine the linear relationship between two variables have a linear relationship or not. Linearity test is used as a condition in which particular parametric regression on the linear regression. One way to test the linearity is Harvey-Collier Test. Results Harvey-Collier Test p value of 0.001. The above results show the linearity assumption is not met, and unknown forms of actual data patterns, it can use one of the methods that the nonparametric regression.

Estimates of bi-responses truncated spline function by 1 to 3 number of knots, has a minimum GCV values presented in Table 1 are as follows, by 1 knots linear for the minimum GCV criteria, and 1 knots qubic by the maximum of R^2. The point of knot in each models can be seen in Table 2. Optimal parameter estimation results are presented in Table 3 and 4.

Table 1. Value of GCV and MSE Truncated spline Bi-responses in Different Number of Knot

Polinomial Order	Number of Knot	Minimum GCV	MSE	R^2
Linear	1	**1.095**	0.045	0.713
	2	1.134	0.039	0.792
Quadratic	1	1.395	0.038	0.807
	2	1.776	0.037	0.816
Qubic	1	1.383	0.038	0.811
	2	1.638	0.030	**0.876**

Table 2. Point of 1 knots

Function	Predictors	Knot in Y_1	Knot in Y_2
Linear 1 knot	x_1	10.987	10.987
	x_2	11.05	11.05
Quadratic 1 knot	x_1	5.783	5.783
	x_2	11.05	11.05
Qubic 1 knot	x_1	5.783	5.783
	x_2	5.783	5.783

Table 3. Parameter Estimation for 1 Knot Linear

No	Parameter	Y_1	Parameter	Y_2
1	α_{10}	-0.002	α_{20}	0.002
2	α_{111}	0.729	α_{211}	0.614
3	β_{111}	-0.566	β_{211}	-0.558
5	α_{121}	-0.026	α_{221}	-0.031
6	β_{121}	1.145	β_{221}	1.195

Table 4. Parameter Estimation for 2 Knot Cubic

No	Parameter	Y_1	Parameter	Y_2
1	α_{10}	-0.093	α_{20}	-0.119
2	α_{111}	0.335	α_{211}	0.453
3	α_{112}	1.044	α_{212}	0.441
4	α_{113}	-0.494	α_{213}	-0.327
5	β_{111}	-0.780	β_{211}	-0.426
6	α_{121}	2.255	α_{221}	2.616
7	α_{122}	-6.738	α_{222}	-7.778
8	α_{123}	2.321	α_{223}	2.614
9	β_{121}	4.300	β_{221}	4.895

Function estimation on linear and qubic truncated Spline bi-responses by 1 point knots based on minimum GCV, can be written into the form of the following equation

Linear

$$\hat{f}_1(x) = -0.002 + 0.729x_1 - 0.566(x_1 - 10.987)_+ - 0.026x_2 + 1.145(x_2 - 11.05)_+$$
$$\hat{f}_2(x) = -0.002 + 0.614x_1 - 0.558(x_1 - 10.987)_+ - 0.031x_2 + 1.195(x_2 - 11.05)_+$$

Qubic

$$\hat{f}_1(x) = -0.93 + 0.335x_1 + 1.044x_1^2 - 0.494x_1^3 - 0.780(x_1 - 5.783)_+^3 + 2.255x_2 - 6.738x_2^2 + 2.321x_2^3 + 4.300(x_2 - 5.783)_+^2$$
$$\hat{f}_2(x) = -0.119 + 0.453x_1 + 0.441x_1^2 - 0.327x_1^3 - 0.4626(x_1 - 5.783)_+^3 - 2.616x_2 - 7.778x_2^2 + 2.614x_2^3 + 4.895(x_2 - 5.783)_+^2$$

The last model selected was a model qubic with the level of determination coefficient R^2 equals 0.876. The coefficient of determination of 87.6%. This means that 87.6% of the value of inflation (Y1) and Economic Growth (X2) is determined by the Minimum Living Needs (X1) and the Regional Minimum Wage (X2), the remaining 28.7% is determined by other factors.

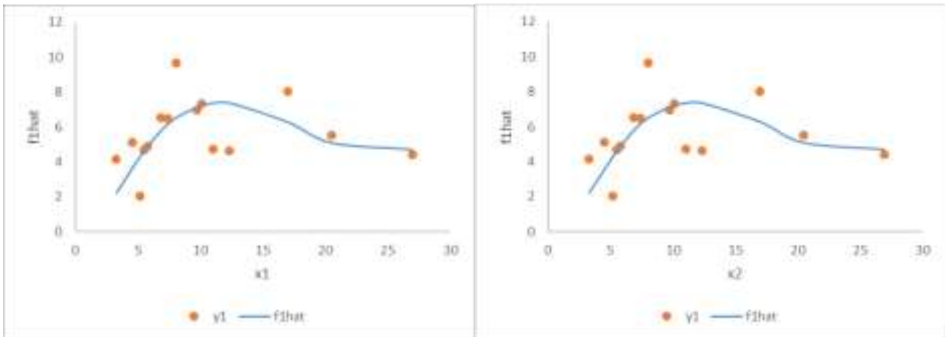

Figure 3 Curve Estimation between Minimum Living Needs (X1) and the Regional Minimum Wage (X2) on Inflation (Y1)

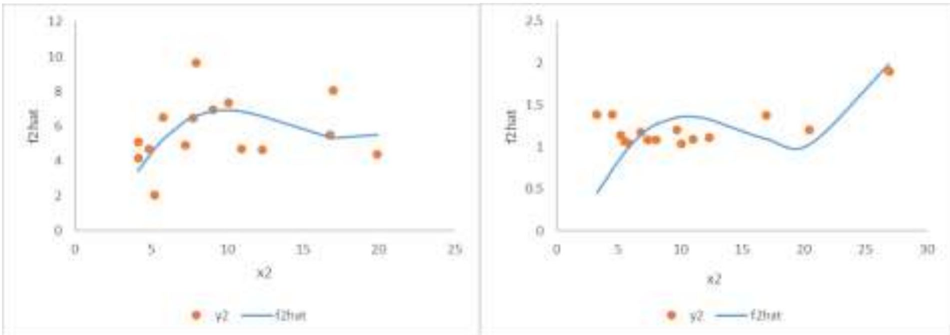

Figure 4 Curve Estimation between Minimum Living Needs (X1) and the Regional Minimum Wage (X2) on Economic Growth (Y2)

4. CONCLUSION

Based on the results and discussion, it can be concluded that the effect of Minimum Living Needs (X1) and the Regional Minimum Wage (X2) on Inflation (Y1) and Economic Growth (X2) can be formed through the spline Truncated in Nonparametric Regression Bi-responses. The coefficient of determination of 87.6%. This means that 87.6% of the value of inflation (Y1) and Economic Growth (X2) is determined by the Minimum Living Needs (X1) and the Regional Minimum Wage (X2), the remaining 28.7% is determined by other factors.

Reccomendation for this research are as follows (1) In this study, use only the 3 maximum number of, future research can be investigate for more than 3 knots. (2) The data in this study is limited to the cross section data, then can be implemented to longitudinal or spatial data.

5. ACKNOWLEDGMENTS

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