IJournals: International Journal of Software & Hardware Research in Engineering
ISSN-2347-4890
Volume 4 Issue 3 March, 2016

Design and Implementation of Classical PI Controller for an Inverted Pendulum

Author: P.Kalaiselvi[1]; M.Anand[2]; L.Kanimozhi[3]

Affiliation: Assistant Professor, SNS College of Technology[1];
Assistant Professor, SNS College of Technology[2];
Assistant Professor, SNS College of Technology[3]

E-mail: kalaiselvipalanivel@gmail.com[1]; anandmeproject@gmail.com[2]; kanikongumts@gmail.com[3]

ABSTRACT

The Inverted Pendulum System is a below actuated, unstable and nonlinear system. Stabilization at the inverted point is accomplished through linear state feedback. Thus, control system design of such a system is a difficult work. The original higher order model of an Inverted Pendulum is reduced to first order model using Pole clustering and Cross Multiplication techniques approximates the transfer function of the original model by retaining the dominant character. For this Unstable First Order Plus Time Delay (UFOPTD) system, the conventional PI controller is designed using compensation technique. The controller's performance is analyzed and reported in terms of error indices and quality indices. Simulation results expose that the conventional PI controller enhance the performance in all aspects when compared to other control techniques. This PI controller is compared with the Majhi & Atherton PI controller.

Keywords: Model Order Reduction, Pole clustering technique, Cross multiplication technique, Inverted Pendulum, PI control

1. INTRODUCTION

The inverted pendulum problem is one of the major problems in control theory and has been studied excessively in control literatures. It is well established standard problem that provides many challenging problems to control design. The inverted pendulum system is nonlinear, unstable, non-minimum phase and under actuated. Because of their nonlinear nature pendulums have maintain their usefulness and they are now used to illustrate many of the ideas emerging in the field of non- linear control. For the most part of the modern technologies use the basic concept of Inverted Pendulum, such as position control of space satellites and rockets, hallway of aircrafts, balancing of ship against tide , Seismometer (which monitors motion of the ground due to underground eruption and nuclear explosions) etc.

An Inverted Pendulum has its mass above the pivoted position, which is mounted on a cart which can be moved horizontally. The pendulum is stable while execution downwards, but the inverted pendulum is inherently unstable and need to be balanced. To achieve this, suitable control theory is required. The Inverted Pendulum is a non-linear time variant open loop system. So the ordinary linear technique cannot model the non-linear dynamics of the system. This makes the system more challenging for analysis. The dynamics of the actual non-linear system is more difficult. But this non-linearized system can be approximated as a linear system if the working region is small, i.e. the variation of the angle from the norm. There are mainly three ways of balancing an inverted pendulum i.e. (i) by applying a torque at the pivoted point (ii) by touching the cart horizontally (iii) by oscillating the support rapidly up and down. Many literature describe the inverted pendulum system modelling and controller design such as Anderson (1989), Wang (1993), Wang (2011), Yadav et al. (2011) and Wu (2011). The transfer function of an inverted pendulum is a higher order system. Controller design for the higher order system is difficult. So the order reduction is must in this process. Model Order reduction is the technique that convert higher order system consisting large number of poles into lower order system containing dominant poles. The low order model which retains the dominant characteristics of the original model.

The exact analysis of high order system is both tedious and costly. Several applications involving signal processing, controlling chemical plants, nuclear reactor and process industries increase the importance of the reduced order model. Various methods have been proposed for the order reduction of continuous and discrete time systems such Chen et al. (1979), Gutman et al. (1982) Various methods for order reduction includes Routh approximation, Moment matching technique, Pade approximation, Aggregation method, Pole clustering techniques, etc. The pade approximation techniques have simple features such as computation simplicity and fitting of the time moments. These are proposed by Ismail et al (1997), Prasad (2003), srinivasan and Krishnan (2010), Ramesh et al. (2009) and Ayyar et al. (2012). A major drawback of this method is that it occasionally leads to an unstable reduced order model. The Classical PI controller is designed for low order model. The most popular tuning formula is Ziegler-Nichols tuning formula which is designed to have quarterly decayed overshoot characteristic in the step response.

In this paper, a new attempt is made to design a classical PI Controller for an Inverted Pendulum. This Classical PI controller is compared with and Mahji and Atherton PI control. The performance of the controllers is analyzed and reported in terms of error indices and quality indices.

2. MATHEMATICAL MODELLING OF AN INVERTED PENDULUM

Every control development starts with a plant modeling. The model of the single inverted pendulum is established and the dynamical equation of the system is derived. The system consists of an inverted pole hinged on a cart which is free to move in the x direction. The Fig 1 shows the free body diagram of the Inverted Pendulum.

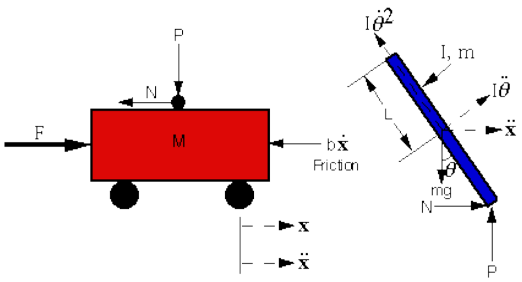

Fig 1: Free body diagram of an Inverted Pendulum

Properties of the cart and pendulum are,

M - Mass of the cart (1.096 Kg)
m - Mass of the pendulum (0.109 Kg)
b - Friction coefficient of the cart (0.1 N/m/sec)
l - Length of the rod (0.25m)
I - Moment of inertia (0.0034 kg m^2)
F - Force acting on the cart
x - Cart position
θ - Angle between the rod and vertically downward direction

There are several methods for finding the dynamics of the Inverted Pendulum system. In this thesis, we have focused on Newton's second law of motion to find the dynamics. For this we need to have a clear idea what forces are acting on each of the free bodies of the system. To get the suitable mathematical model for an Inverted Pendulum system, consider the figure 1.

Adding all forces on the cart in the horizontal direction,

$$M\ddot{x} + b\dot{x} + N = F \qquad (1)$$

Adding all forces on the pendulum rod in the horizontal direction,

$$m\ddot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta = N \qquad (2)$$

Substituting equation (2) in equation (1)

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta = \mathrm{F} \qquad (3)$$

Adding all the forces along the vertical direction f the pendulum,

$$Psin\theta + Ncos\theta - mgsin\theta = ml\ddot{\theta} + m\ddot{x}cos\theta \qquad (4)$$

Considering sum of the moments about the centre of gravity (C.G) of the pendulum,

$$-Plsin\theta - Nlcos\theta = I\ddot{\theta} \qquad (5)$$

From the equations (4) and (5)

$$(I + ml^2)\ddot{\theta} + mglsin\theta = -ml\ddot{x}cos\theta \qquad (6)$$

The real time system i.e. an inverted pendulum is a non-linear system. For simplicity of modelling and simulation, we have to take a small case approximation such that the system will be linear one. Where $\theta = \pi$ or $\theta = \pi + \varphi$.

$$cos\theta = -1,\ sin\theta = -\varphi,\ \frac{d\theta}{(dt)^2} = 0$$

After the linearization equation (6) becomes,

$$(l + ml^2)\ddot{\varphi} - mgl\varphi = ml\ddot{x} \quad (7)$$

And equation (3) becomes,

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\varphi} = F \quad (8)$$

Equation (8) becomes,

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\varphi} = u \quad (9)$$

To derive the transfer function of an inverted pendulum, take Laplace Transform of equation (7),

$$(l + ml^2)\phi(s)s^2 - mgl\phi(s) = mlX(s)s^2 \quad (10)$$

Laplace Transform of equation (9) is

$$(M + m)X(s)s^2 + bX(s)s - ml\phi(s)s^2 = U(s) \quad (11)$$

Solving equation (10) for getting X(s)

$$X(s) = \left[\frac{(l+ml^2)}{ml} - \frac{g}{s^2}\right]\phi(s) \quad (12)$$

Substituting equation (12) in (11)

$$(M + m)\left[\frac{(l+ml^2)}{ml} - \frac{g}{s^2}\right]\phi(s)s^2 + b\left[\frac{(l+ml^2)}{ml} - \right.$$
$gs2\phi ss{-}ml\phi ss2{=}Us$ (13)

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 - \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \quad (14)$$

Where $q = [(M + m)(I + ml^2) - (ml)^2]$

In equation (14), it is clear that one pole and zero is at origin. This leads to cancelation of one pole and zero. So resulting equation will be,

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 - \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \quad (15)$$

Here in this case the angle from the vertical position (ϕ(s)) is taken as the output and the applied force to the cart (U(s)) is taken as the input function. The transfer function of pendulum angle and external force acting on the cart, after applying the parameter values actual system model will be obtained.

$$\frac{\phi(s)}{U(s)} = \frac{2.35655s}{s^3 + 0.0883167s^2 - 27.9169s - 2.30942} \quad (16)$$

3. MODEL ORDER REDUCTION

The transfer function of pendulum angle and external force acting on the cart, after applying the parameter values are

$$\frac{\phi(s)}{U(s)} = \frac{2.35655s}{s^3 + 0.0883167s^2 - 27.9169s - 2.30942} \quad (17)$$

The above transfer function is a higher order system i.e. in 3rd order. Here the MOR method is used to reduce the order of the system. The pole clustering technique is used to get a second order model. This will be reduced into first order by using Cross multiplication technique.

$$p_c = \left\{\left(\sum_{i=1}^{k}\left(\frac{1}{p_i}\right)\right) \div k\right\}^{-1}$$

The poles are +5.2809, -5.2865, -0.0827

$$\frac{\phi_{r2}(s)}{U_{r2}(s)} = \frac{0.4323s}{s^2 - 5.118s - 0.852} \quad (18)$$

The second order transfer function is further reduced to first order by using Cross multiplication technique.

$$\frac{0.4323s}{s^2 - 5.118s - 0.852} = \frac{d_0}{e_1 s + e_0}$$

$$\frac{\phi_{r1}(s)}{U_{r1}(s)} = \frac{0.072}{0.161s - 0.852} \quad (19)$$

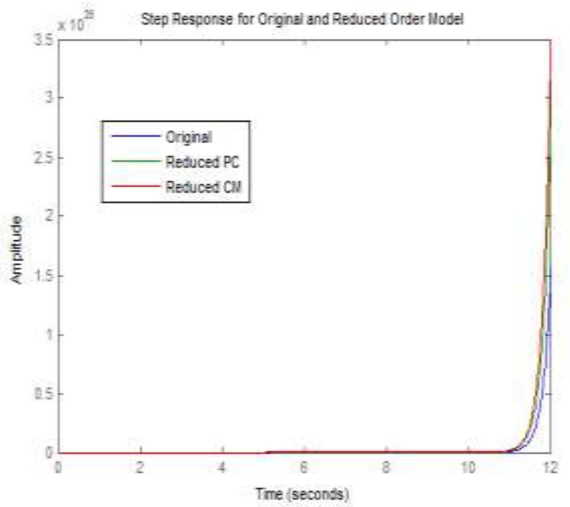

Fig 2: Step response of original and reduced order model inverted pendulum

The step response of original and reduced order model inverted pendulum through pole clustering and cross multiplication method is as shown in Fig 2. The reduced order model, which retains and reflects the important characteristics of the original system as close as possible.

4. CLASSICAL PI CONTROLLER DESIGN

4.1 Controller design using Compensator technique

Compensator is an additional device or component used to improve the system performance. It is used to alter the response of a system in order to accommodate the set region criteria. The performance is improved by adding additional poles and zeros to the system.

The Compensation is mainly required in the following situations,

1. When a system is absolutely unstable, then compensation is required to stabilize the system and also to gather the desired performance.
2. When the system is stable, compensation is provided to obtain the desired performance.

4.2 Inverted pendulum without compensation

The Fig 3 shows the step response of open loop uncompensated Inverted Pendulum system. Here also, theta diverges very rapidly as the system is highly unstable. The runaway nature of the response indicates instability.

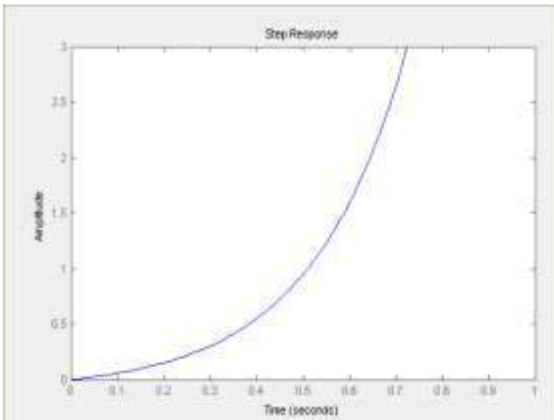

Fig 3: Step response of open loop uncompensated Inverted Pendulum system

Many systems are unstable in open loop configuration but stable in closed loop configuration. The other way round is also probable that the system is stable in the open loop but unstable in closed loop, although this case is rare. The closed loop uncompensated system can be studied by screening the root locus plot of the system.

4.3 Inverted pendulum with compensation

The Fig 4 shows the step response of open loop uncompensated Inverted Pendulum system. In this Compensation, we have use the Root Locus Technique because they permit accurate computation of the time-domain response in addition to yielding readily available frequency response information. The system analysis indicates that using only the gain compensation in closed loop cannot control the IP. Reshaping of the system root locus is necessary so that for definite range of gains, the system has all its roots in the left half plane (stable region) of the s-plane. The given system is unstable for all values of gain, so the root locus must be reshaped so that the part of each branch spray in the left half s-plane, thereby making the system stable.

Also the preferred performance specifications established for the system must be achieved. The design specification of the closed loop system is 5 % of settling time and zero steady state error. The compensation of the system by the introduction of poles and zeroes is used to develop the operating performance.

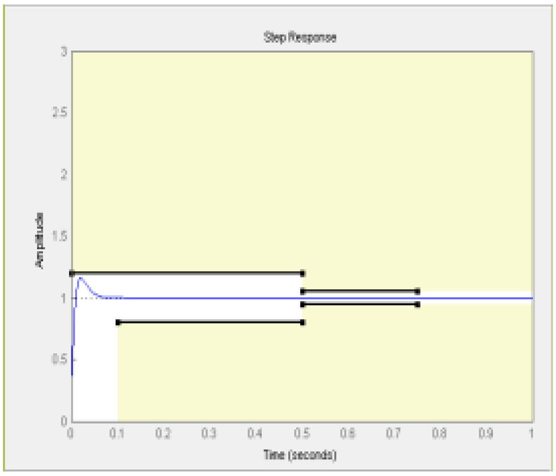

Fig 4: Step response of compensated Inverted Pendulum system

However, every additional compensator pole increases the number of roots of the closed-loop characteristics equation. The compensator designing has been done with MATLAB SISO DESIGN TOOL.

The DC Gain of the Closed-Loop Compensated Inverted Pendulum System is 19096. The settling time is 0.059 sec (less than 0.5 sec). The percentage overshoot is 15.3%. The steady-state error is zero.

$$PI = K_C * K_P + \frac{K_I}{s} \tag{20}$$

K_P= Proportional gain
K_I= Integral gain

The compensator equation is found to be:

$$G_C = 19096 * \left(\frac{0.022s+1}{s}\right) \tag{21}$$

Comparing it with following equation,

$$G_C = K_C * \left(\frac{K_P s + K_I}{s}\right)$$

KC = 19096 ; Kp = 0.022 ; Ki = 1 (22)

5. MAJHI & ATHERTON PI CONTROLLER DESIGN

PI controller is designed using Majhi & Atherton tuning rule for Unstable First Order Plus Time Delay (UFOPTD) system.

Majhi& Atherton PI tuning rule

$$K_C = \frac{1}{K_m}\left(0.889 + \frac{e^{-\frac{\tau_m}{T_m}} - 0.064}{e^{\frac{\tau_m}{T_m}} - 0.990}\right) \tag{23}$$

$$T_i = \frac{2.6316 T_m \left(e^{\frac{\tau_m}{T_m}} - 0.966\right)}{e^{-\frac{\tau_m}{T_m}} - 0.377} \tag{24}$$

From that above PI tuning value, Kp and Ki values are obtained. They are

KC = 301.282
Ki = 5954.18 (25)

The performance of the classical PI controller using compensation technique is analyzed and compared with the Majhi & Atherton PI controller through simulation studies. For analysis, the structure for an Inverted Pendulum system is modeled using MATLAB/SIMULINK shown in Fig 5.

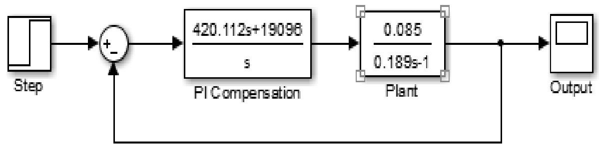

Fig 5: Model of Inverted Pendulum with classical PI Controller

6. RESULT AND DISCUSSION

To evaluate the performance of the PI, Majhi & Atherton PI controllers, closed loop simulations are carried out for the set point. Closed loop simulated transient responses are recorded in Figures 6.

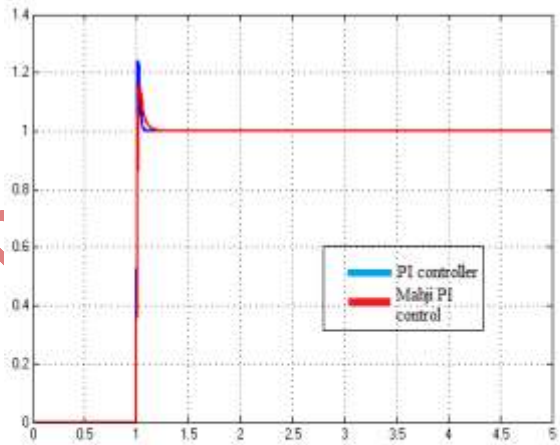

Fig 6: Performance result

The performance of the controllers is analyzed and reported in terms of performance dealings Integral Squar Error (ISE), Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE) and settling time (ts) of output signal in table 1.

Table 1. UFOPTD Performance measures for step response

Tuning Method	ISE	IAE	ITAE	T_s (sec)
PI Controller	0.6977	2.8887	15.3989	35
Majhi and Atherton PI Control	0.7865	3.4592	25.3704	49

From the simulation results in table 1, it is clear that the proposed classical PI controller enhances the performance in all aspects when compared to the

Majhi and Atherton PI control technique. The PIcontrol strategy is developed for Unstable First Order Plus Time Delay (UFOPTD) process i.e. Inverted Pendulum System.

7. CONCLUSION

The present work proposed a Classical PI control strategy for an Inverted Pendulum process. The model order reduction techniques suggested in this paper give better approximated reduced order model for the given Inverted Pendulum system. Because of this, the reduced order system performance as close as probable to the higher order system response is obtained. The Inverted Pendulum system is modeled as UFOPTD transfer function. Classical PI controller is developed for an Inverted Pendulum and the simulation studies were analyzed by ISE, IAE and ITAE. It is clear that that the Classical PI control strategy yields a fair transient response when compared to other controller technique.

REFERENCES

[1] Anderson, C. W. (1989). Learning to control an inverted pendulum using neural networks. Control Systems Magazine, IEEE, 9(3), 31-37.

[2] Ayyar, K., Gurusamy, G. and Thanushkodi, K. (2012). "Design of speed controller for speed control of converter fed DC motor using Model Order Reduction Technique", European Journal of Scientific Research, Vol. 76, No.2, pp.288-299.

[3] Chen, T. C., Chang, C.Y. and Han, K.W. (1979). "Reduction of Transfer functions by the stability equation method", Journal of Franklin Inst, Vol.308, pp 389 – 404.

[4] Fu, H. L., & Wu, C. L. (2011). The inverted pendulum error analysis with MATLAB simulation and validation. In Multimedia Technology (ICMT), 2011 International Conference on (pp. 6603-6606). IEEE.

[5] Ismail, O., Bandyopadhyay, B. and Gorez, R. (1997). "Discrete interval system reduction using Pade approximation to allow retention of dominant poles", IEEE Trans. on Cir. Sys., Vol. 44, No. 11, pp. 1075-1078.

[6] Manigandan, T., Devarajan, N. and Sivanandam, S.N. (2005). "Design of PID Controller using Reduced Order Model", Academic Open Internet Journal, Vol.5.

[7] Gutman, Carl Fredrik mannerfelt and Permolander. (1982). "Contributions to the model reduction problem", IEEE Transaction on Automatic control, Vol.AC-27, pp.454-455.

[8] Prasad, R. Sharma, S.P. and Mittal, A.K. (2003). "Improved Pade approximants for multivariable systems using stability equation method," J. Inst. Engg. IE (I) - EL, Vol. 86, pp. 161–165.

[9] Ramesh, K., Nirmalkumar, A. and Gurusamy, G. (2009). "Design of Digital IIR filters with the Advantages of Model Order Reduction Technique", International Journal of Electrical and Computer Engineering, Vol.4, No.14, pp.903-908.

[10] Shamash, Y. (1974). "Stable reduced order models using Pade type approximations", IEEE Transactions on Automatic Control, Vol.19, No.5, pp.615-616.

[11] Srinivasan, M., & Krishnan, A. (2010). Transformer linear section model order reduction with an improved pole clustering. European Journal of Scientific Research, 44(4), 541-549.

[12] Wang, L. X. (1993). Stable adaptive fuzzy control of nonlinear systems. Fuzzy Systems, IEEE Transactions on, 1(2), 146-155.

[13] Wang, J. J. (2011). Simulation studies of inverted pendulum based on PID controllers. Simulation Modelling Practice and Theory, 19(1), 440-449.

[14] Yadav, A. K., Gaur, P., Mittal, A. P., &Anzar, M. (2011, January). Comparative analysis of various control techniques for inverted pendulum. In Power Electronics (IICPE), 2010 India International Conference on (pp. 1-6). IEEE.