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Radiation and Viscous Dissipation Effects on MHD Heat and Mass Flow of a Fluid with Temperature Dependent Viscosity Inclined in a Porous Medium under the Influence of Temperature Dependent Suction Velocity.

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Abstract

In this analysis, MHD flow of a variable viscous fluid past a continuously moving vertical porous plate under temperature dependent suction velocity was studied. A time-dependent suction was assumed and the radiative heat flux was described using Rosseland approximation. A two-stage asymptotic series expansion about small parameters ε and δ was used to simplify and solve analytically the governing equations. Following, some important features of the flow for different values of the governing parameters were analysed, discussed and presented through tables and graphs.

Keywords: *variable viscosity, unsteady, MHD, radiative heat flux, chemical reaction, suction velocity.*

1.0 terature review

Numerous challenges of heat and mass transfer along a vertical porous plate related to geophysics, astrophysics, petroleum, chemical and bio-chemical are subjects of considerable attention to researchers. Real-life encounters in nature and industries involving: the flow of oil through porous rock, the filtration of solid from liquid, flow of liquids through ion exchange beds, permeation of drug through human glands, the extraction of geo-thermal energy from the beneath of the earth crust to the surface layers and application chemical reactor for separation or purification of mixtures flow through porous medium are common occurrences that have assumed substantial importance.

Due to those aforementioned benefits of heat and mass transfer, plentiful mathematical models related to all such situations have been formulated and analysed in detail by researchers. Ibrahim and Makinde [1] investigated a two-dimensional, steady, viscous, incompressible, electrically conducting and laminar free convection boundary layer flow with radiation from a flat plate in a chemically reactive medium in the presence of a transverse magnetic field. The problem was solved using shooting techniques with the fourth order Runge-Kutta integration scheme. Muhammed and Shahzard [2] investigated radiation effects on MHD two dimensional stagnation point flow of a steady viscous incompressible electrically conducting fluid towards a shrinking sheet in the presence of a transverse magnetic field. The governing equation together with the associated boundary conditions are first reduced to non-linear ordinary differential equations and then solved by a method based on finite difference discretization. Das [3], in his work, researched on the unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous flat plate with suction when the plate accelerates in its own plane. The governing equations were solved both analytically and numerically using finite difference scheme.

All the aforementioned works on MHD flow of a viscous fluid had been confined to a case of constant viscosity. However, it is known that this physical property may change significantly with temperature, time or space variable. Gary et al [4], and Mehta and Sood [5] have established that there is wide disparity in flow characteristics of variable viscosity and that of constant. Based on this, Hosain et al [6-8], Geetha and Moorthy [9] and Ziya and Manoj [10] have considered works on MHD flow of a variable viscosity. They all assumed the viscosity to be inversely proportional to linear function of temperature. In another development, Okedoye, and Farayola [11] and Mostafa [12], in their works on MHD flow of a variable viscosity assumed that viscosity obeyed the Reynold model. All the above references [4-12] dealt with constant suction and the solutions were obtained numerically.

Prakash and Ogulu [13] investigated unsteady two-dimensional flow of a radiating and chemically reacting MHD fluid with time-dependent suction. Shanker [14] studied the effects of thermal radiation, time-dependent suction and chemical reaction on the two-dimensional flow of an incompressible. In this present work, the effects of thermal radiation, temperature dependent viscosity and chemical reaction on the heat and mass transfer in MHD flow of an optically tick viscous, incompressible, electrically conducting fluid past an infinite vertical plate in a porous medium under temperature dependent suction velocity had been made.

1.1 Nomenclature

u : Velocity along x coordinate	T' : Non dimensional fluid temperature
v : Velocity along y coordinate	C' : Non dimensional species concentration
g : Acceleration due to gravity	T : Fluid temperature
U' : Non dimensional fluid velocity	γ : Reaction parameter
T_w : Ambient temperature	σ^* : Stefan- Boltzmann constant
C : Species concentration	B : Coefficient of mass expansion
C_w : Ambient species concentration	B_τ : Coefficient of thermal expansion
B_0 : Transverse magnetic field	ρ_w : Ambient density
τ : Skin-friction coefficient	σ : Electrical conductivity
A : Suction parameter	ρ : Density of the fluid
k : Thermal conductivity	Sc : Schmidt number
c_p : Specific heat at constant pressure	R : Radiation parameter
q_r : Radiative heat flux	G_{rc} : Mass grashof number
v_0 : Normal velocity at the plate	$G_{r\tau}$: Thermal grashof number
k^* : Mean absorption coefficient	δ : Delta, $0 \le \delta_o << 1$
M : Hartmann number	ε : Epsilon, $0 \le \varepsilon_o << 1$
Nu : Nuselt number	Sh : Sherwood number
n : Angular velocity	t : Time
$\Pr$: Prandtl number	μ : Fluid viscosity
D : Molar diffusivity	A^* : Pre-exponential factor

2.0 Mathematical Formulation

This analysis consider a MHD flow of a viscous, incompressible, electrically conducting fluid over an infinite vertical plate in a porous medium under suction velocity .The direction of the flow is in x- axis along the plate and y- axis normal to it. A uniform magnetic field is applied normal to the direction of the flow. The magnetic Reynold number is assumed to be less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Also, all the fluid properties are assumed to be constant except that of the influence of density variation with temperature. So, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium. Initially at $t \le 0$, the plate as well as fluid is assumed to be at the same temperature and the concentration of species is very low so that the Soret and Dofour effect are neglected [15]. When $t > 0$, the temperature of the plate is instantaneously raised (or lowered) to $T_w^{'}$ and the concentration of species is raised (or lowered) to $C_w^{'}$. Under the above assumptions and taking the usual Boussinesq's approximation into account, the governing equations for momentum, energy and concentration are presented below:

$$\frac{dv^{'}}{dy^{'}} = 0 \tag{2.1}$$

$$\frac{\partial U^{'}}{\partial t^{'}} + v^{'} \frac{\partial U^{'}}{\partial y^{'}} = \frac{1}{\rho_\infty} \frac{\partial}{\partial y^{'}} \left[\mu \frac{\partial U^{'}}{\partial y^{'}} \right] + g\beta^* \left(T^{'} - T_\infty\right) + g\beta\left(C^{'} - C_\infty\right) - \frac{\sigma B_0^2 U^{'}}{\rho_\infty} \tag{2.2}$$

$$\frac{\partial C^{'}}{\partial t^{'}} + v^{'} \frac{\partial C^{'}}{\partial y^{'}} = D \frac{\partial^2 C^{'}}{\partial y^{'2}} - A^* \left[C^{'} - C_\infty\right] \tag{2.3}$$

$$\frac{\partial T^{'}}{\partial t^{'}} + v^{'} \frac{\partial T^{'}}{\partial y^{'}} = \frac{k_t}{\rho_\infty c_p} \frac{\partial^2 T^{'}}{\partial y^{'2}} - \frac{1}{\rho_\infty c_p} \frac{\partial q_r}{\partial y^{'}} \tag{2.4}$$

The boundary conditions:

$$U^{'} = v_0 \quad T^{'} = \left(1 + \varepsilon e^{nt}\right)\left[\left(T_w - T_\infty\right) + T_\infty\right] \quad C^{'} = \left(1 + \varepsilon e^{nt}\right)\left[\left(C_w - C_\infty\right) + C_w\right] \quad \text{at } y^{'} = 0$$

$$U^{'} \to 0, \qquad T^{'} \to T_\infty \qquad C^{'} \to C_\infty \qquad \text{as } y^{'} \to \infty \tag{2.5}$$

From equation (2.1), we take $v^{'} = -v_0\left(1 + \varepsilon A e^{nt}\right)$ where $\varepsilon A << 1$, the minus sign indicates that the suction is towards the plate

By using Rosseland approximation q_r takes the form [16]

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{dT^4}{dy} \tag{2.6}$$

The temperature difference within the fluid assumed sufficiently small such that T^4 may be expressed as a linear function of the temperature. Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, we have

$$T^4 = 4T_\infty{}^3 T - 3T_\infty{}^4 \tag{2.7}$$

Substituting equation (3.7) into (3.6), we obtain

$$q_r = -\frac{16\sigma^* T_\infty^3 d^2T}{3k^* dy^2} \tag{2.8}$$

Using the following non dimensional quantities:

$$y = \frac{v_0 y'}{\vartheta} \qquad U = \frac{U'}{v_0} \qquad \theta = \frac{T' - T_\infty}{T_w - T_\infty} \qquad C = \frac{C' - C_\infty}{C_w - C_\infty}$$

$$t = \frac{v_0{}^2 t'}{\vartheta} \qquad G_{r\tau} = \frac{\vartheta g \beta^* (T_w - T_\infty)}{v_0{}^3} \qquad G_{rc} = \frac{\vartheta g \beta (C_w - C_\infty)}{v_0{}^3} \qquad M = \frac{B_0}{v_0}\sqrt{\frac{\sigma\vartheta}{\rho}}$$

$$\Pr = \frac{\mu_\infty c_p}{k_t} \qquad R = \frac{16\sigma T_\infty{}^3}{3k^* k} \qquad Sc = \frac{\vartheta}{D} \qquad \gamma = A^* \frac{\mu_\infty}{v_0{}^2 \rho_\infty} \qquad n = \frac{4\vartheta n'}{v_0{}^2}$$

on equation (2.2)-(2.5), the dimensionless governing equations for momentum, energy and concentration and their boundary conditions are:

$$\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - \gamma C \tag{2.9}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \theta}{\partial y} = \frac{1+R}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{2.10}$$

$$\frac{\partial U}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{\mu_\infty}\frac{\partial U}{\partial y}\right] + G_{r\tau}\theta + G_{rc}C - M^2 U \tag{2.11}$$

Where the parameters $\varepsilon, G_{r\tau}, G_{rc}, k, \Pr, R, \gamma, Sc, and M$ were defined in nomenclature.

The corresponding boundary conditions are

$$U = 1 \qquad \theta = 1 + \varepsilon e^{nt} \qquad C = 1 + \varepsilon e^{nt} \qquad \text{on } y = 0 \tag{2.12}$$

$$U(y) \to 0 \qquad \theta \to 0, \qquad C \to 0 \qquad \text{as } y \to \infty$$

The fluid viscosity $\mu(\theta)$ was assumed to obey the Reynolds model [17]

$$\frac{\mu}{\mu_\infty} = e^{-\alpha\theta} \tag{2.13}$$

Where α, is a parameter depending on the nature of the fluid. Using equation (2.13) in equation (2.11), we obtain

$$\frac{\partial U}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left[e^{-\alpha\theta}\frac{\partial U}{\partial y}\right] + G_{r\tau}\theta + G_{rc}C - M^2 U \tag{2.14}$$

3.0 Method of Solution

To solve equations (2.9), (2.10) and (2.14) we make use of the following transformation

$$U(y,t) = U_0(y) + \varepsilon U_1(y)e^{nt} + 0(\varepsilon^2) + \ldots.. \tag{3.1a}$$

$$\theta(y,t) = \theta_0(y) + \varepsilon\theta_1(y)e^{nt} + 0(\varepsilon^2) + \ldots.. \tag{3.1b}$$

$$C(y,t) = C_0(y) + \varepsilon C_1(y)e^{nt} + 0(\varepsilon^2) + \ldots.. \tag{3.1c}$$

Corresponding to the species equation we have

$$\frac{d^2C_0}{dy^2} + Sc\frac{dC_0}{dy} - \gamma Sc C_0 = 0 \tag{3.2}$$

$C_0 = 1$ as $y = 0$ $C_0 \to 0$ as $y \to \infty$

$$\frac{d^2C_1}{dy^2} + Sc\frac{dC_1}{dy} - (\gamma + n)ScC_1 = -ScA\frac{dC_0}{dy} \quad (3.3)$$

$C_1 = 1$ on $y = 0$ $C_1 \to 0$ as $y \to \infty$

Corresponding to the energy equation we have

$$\frac{d^2\theta_0}{dy^2} + h\frac{d\theta_0}{dy} = 0 \quad \text{where} \quad h = \frac{P_r}{1+R} \quad (3.4)$$

$\theta_0 = 1,$ as $y = 0$ $\theta_0 \to 0,$ as $y \to \infty$

$$\frac{d^2\theta_1}{dy^2} + h\frac{d\theta_1}{dy} - nh\theta_1 = -hA\frac{d\theta_0}{dy} \quad \text{where} \quad h = \frac{P_r}{1+R} \quad (3.5)$$

$\theta_1 = 1,$ on $y = 0$ $\theta_1 \to 0,$ as $y \to \infty$

Corresponding to the momentum equation we have

$$\frac{\partial}{\partial y}\left[e^{-\alpha\theta}\frac{dU_0}{dy}\right] + \frac{dU_0}{dy} - M^2U_0 = -G_{r\tau}\theta_0 - G_{rc}C_0 \quad (3.6)$$

$U_0 = 1$ as $y = 0$ $U_0 \to 0$ as $y \to \infty$

$$\frac{\partial}{\partial y}\left[e^{-\alpha\theta}\frac{dU_1}{dy}\right] + \frac{dU_1}{dy} - nU_1 - M^2U_1 = -G_{r\tau}\theta_1 - G_{rc}C_1 - A\frac{dU_0}{dy} \quad (3.7)$$

$U_1 = 0$ on $y = 0$ $U_1 \to 0$ as $y \to \infty$

Solving equations (3.2)-(3.5) and substitute the results into (3.1b) and (3.1c), we have

$$C(y) = e^{my} + \varepsilon\left[a_3 e^{\beta y} - a_5 e^{my}\right]e^{nt} \tag{3.8}$$

$$\theta(y,t) = e^{qy} + \varepsilon\left[a_8 e^{xy} - a_{10} e^{qy}\right]e^{nt} \tag{3.9}$$

Where

$$\beta = -\frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4(\gamma + n)Sc}\right] \qquad m = -\frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4\gamma Sc}\right] \qquad a_5 = \frac{S_c Am}{\left(m^2 + S_c m - (\gamma + n)S_c\right)}$$

$$a_3 = 1 + a_5 \quad q = -h \qquad x = -\frac{1}{2}\left[h + \sqrt{h^2 + 4\gamma nh}\right] \qquad a_{10} = \frac{hAq}{\left(q^2 + hq - nh\right)} \qquad a_8 = 1 + a_{10}$$

To solve equations (3.6)-(3.7), we make use of the following transformation:

Let

$$\alpha = o(\delta) \text{ where } \delta << 1 \tag{3.10}$$

$$U_0 = U_{00} + \delta U_{01} + \ldots + h..o.t \tag{3.11}$$

$$U_1 = U_{10} + \delta U_{11} + \ldots + h..o.t \tag{3.12}$$

Substitute equations (3.10)-(3.12) into equations (3.6)-(3.7) and compile the order of δ.

We have

$$\frac{d^2U_{00}}{dy^2} + \frac{dU_{00}}{dy} - M^2 U_{00} = -G_{r\tau}\theta_0 - G_{rc}C_0 \tag{3.13}$$

$$U_{00}(y) = 1 \text{ as } y \to 0 \qquad\qquad U_{00}(\infty) = 0 \quad \text{ as } y \to \infty$$

$$-\frac{d}{dy}\left(\theta\frac{dU_{00}}{dy}\right)+\frac{d^2U_{01}}{dy^2}+\frac{dU_{01}}{dy}-M^2U_{01}=0 \tag{3.14}$$

$$U_{01}(0)=0 \qquad U_{01}(\infty)=0$$

$$\frac{d^2U_{10}}{dy^2}+\frac{dU_{10}}{dy}-\left(n+M^2\right)U_{10}=-G_{r\tau}\left(a_8e^{-xy}+a_{10}e^{-qy}\right)-G_{rc}\left(a_3e^{-\beta y}+a_5e^{-my}\right)- A\left(-\lambda a_{11}e^{-\lambda y}-qa_{13}e^{-qy}-ma_{14}e^{-my}\right) \tag{3.15}$$

$$U_{10}(0)=0 \qquad U_{10}(\infty)=0$$

$$\frac{d^2U_{11}}{dy^2}-\frac{d}{dy}\theta\frac{dU_{10}}{dy}+\frac{dU_{11}}{dy}-\left(n+M^2\right)U_{11}= -A\begin{pmatrix}-\eta a_{15}e^{-\eta y}-(\lambda+x)a_{17}e^{-(\lambda+x)y}-(q+x)a_{18}e^{-(q+x)y}-(m+x)a_{19}e^{-(m+x)y}-(\lambda+q)a_{20}e^{-(\lambda+q)y}-2qa_{21}e^{-2qy}\\-(m+q)a_{22}e^{-(m+q)y}\end{pmatrix} \tag{3.16}$$

$$U_{11}(0)=0 \qquad U_{11}(\infty)=0$$

Solving equations (3.13)-(3.16) and substitute the results into (3.1a), we get

$$U(y,t)=a_{11}e^{\lambda y}-a_{13}e^{qy}-a_{14}e^{my}+ \delta\left[a_{15}e^{\eta y}+a_{17}e^{(\lambda+x)y}-a_{18}e^{(q+x)y}-a_{19}e^{(m+x)y}+a_{20}e^{(\lambda+q)y}+a_{21}e^{2qy}+a_{22}e^{(m+q)y}\right] \varepsilon\left[\begin{matrix}a_{23}e^{ry}-a_{25}e^{xy}+a_{26}e^{qy}-a_{27}e^{\beta y}+a_{28}e^{my}-a_{29}e^{\lambda y}+\\ \delta\left[\begin{matrix}a_{30}e^{sy}+a_{32}e^{(r+q)y}-a_{33}e^{(q+x)y}+a_{34}e^{2qy}-a_{35}e^{(\beta+q)y}+a_{36}e^{(m+q)y}-a_{37}e^{(\lambda+q)y}+a_{38}e^{(r+x)y}\\-a_{39}e^{2xy}-a_{40}e^{(\beta+x)y}+a_{41}e^{(m+x)y}-a_{42}e^{(\lambda+x)y}-a_{43}e^{\eta y}\end{matrix}\right]\end{matrix}\right]e^{nt} \tag{3.17}$$

Where

$$\eta=-\frac{1}{2}\left[1+\sqrt{1+4M^2}\right] \quad q=-h \qquad \lambda=-\frac{1}{2}\left[1+\sqrt{1+4M^2}\right] \quad m=-\frac{1}{2}\left[Sc+\sqrt{Sc^2+4\gamma Sc}\right]$$

$$r=-\frac{1}{2}\left[1+\sqrt{1+4\left(n+M^2\right)}\right] \quad q=-h \qquad \beta=-\frac{1}{2}\left[Sc+\sqrt{Sc^2+4(\gamma+n)Sc}\right]$$

$$s=-\frac{1}{2}\left[1+\sqrt{1+4\left(n+M^2\right)}\right] \quad x=-\frac{1}{2}\left[h+\sqrt{h^2+4\gamma nh}\right] \qquad a_{13}=\frac{G_{r\tau}}{\left(q^2+q-M^2\right)}$$

$$a_{14}=\frac{G_{rc}}{\left(m^2+m-M^2\right)} \quad a_{11}=1+a_{13}+a_{14} \qquad a_{17}=\frac{\varepsilon(\lambda+x)\lambda a_{11}a_8e^{nt}}{\left((\lambda+x)^2+(\lambda+x)-M^2\right)}$$

$$a_{18}=\frac{\varepsilon(q+x)qa_{13}a_8e^{nt}}{\left((q+x)^2+(q+x)-M^2\right)} \qquad a_{19}=\frac{\varepsilon(m+x)ma_{14}a_8e^{nt}}{\left((m+x)^2+(m+x)-M^2\right)}$$

$$a_{20}=-\frac{\left[\lambda a_{11}-\varepsilon\lambda a_{11}a_{10}e^{nt}\right](\lambda+q)}{\left((\lambda+q)^2+(\lambda+q)-M^2\right)} \quad a_{21}=\frac{\left[\varepsilon a_{10}e^{nt}-1\right]2q^2a_{13}}{\left(4q^2+2q-M^2\right)} \quad a_{22}=\frac{\left[\varepsilon ma_{14}a_{10}e^{nt}-ma_{14}\right](m+q)}{\left((m+q)^2+(m+q)-M^2\right)}$$

$$a_{15}=-a_{17}+a_{18}+a_{19}-a_{20}-a_{21}-a_{22} \quad a_{25}=\frac{G_{r\tau}a_8}{\left(x^2+x-\left(n+M^2\right)\right)} \qquad a_{26}=\frac{\left(G_{r\tau}a_{10}+Aqa_{13}\right)}{\left(q^2+q-\left(n+M^2\right)\right)}$$

$$a_{27}=\frac{G_{rc}a_3}{\left(\beta^2+\beta-\left(n+M^2\right)\right)} \qquad a_{28}=\frac{\left(G_{rc}a_5+Ama_{14}\right)}{\left(m^2+m-\left(n+M^2\right)\right)} \qquad a_{29}=\frac{A\lambda a_{11}}{\left(\lambda^2+\lambda-\left(n+M^2\right)\right)}$$

$$a_{23}=a_{25}-a_{26}+a_{27}-a_{28}+a_{29} \qquad a_{32}=\frac{\left[r+q-\varepsilon a_{10}re^{nt}-\varepsilon a_{10}qe^{nt}\right]ra_{23}}{\left((r+q)^2+(r+q)-\left(n+M^2\right)\right)}$$

$$a_{33}=\frac{\left[x^2a_{25}+qxa_{25}-\varepsilon a_{10}x^2a_{25}e^{nt}-\varepsilon a_8q^2a_{26}e^{nt}-\varepsilon qa_8xa_{26}e^{nt}-\varepsilon qa_{10}xa_{25}e^{nt}-A(q+x)a_{18}\right]}{\left((x+q)^2+(x+q)-\left(n+M^2\right)\right)}$$

$$a_{34}=\frac{\left[a_{26}q-\varepsilon a_{26}a_{10}qe^{nt}-Aa_{21}\right]2q}{\left(4q^2+2q-\left(n+M^2\right)\right)} \qquad a_{35}=\frac{\left[\beta+q-\varepsilon a_{10}\beta e^{nt}-\varepsilon a_{10}qe^{nt}\right]\beta a_{27}}{\left((\beta+q)^2+(\beta+q)-\left(n+M^2\right)\right)}$$

$$a_{36}=\frac{\left[m^2a_{28}+qma_{28}-\varepsilon a_{10}m^2a_{28}e^{nt}-\varepsilon a_{10}qma_{28}e^{nt}-A(m+q)a_{22}\right]}{\left((m+q)^2+(m+q)-\left(n+M^2\right)\right)}$$

$$a_{37}=\frac{\left[\lambda^2 a_{29}+q\lambda a_{29}-\varepsilon a_{10}\lambda^2 a_{29}e^{nt}-\varepsilon a_{10}q\lambda a_{29}e^{nt}+A(\lambda+q)a_{20}\right]}{\left((\lambda+q)^2+(\lambda+q)-(n+M^2)\right)}\quad a_{38}=\frac{\left[a_8 re^{(r+x)y}+a_8 xe^{nt}\right]\varepsilon ra_{23}e^{nt}}{\left((r+x)^2+(r+x)-(n+M^2)\right)}$$

$$a_{39}=\frac{2\varepsilon a_8 x^2 a_{25}e^{nt}}{\left(4x^2+2x-(n+M^2)\right)}\quad a_{40}=\frac{[\beta+x]\varepsilon a_8\beta a_{27}e^{nt}}{\left((\beta+x)^2+(\beta+x)-(n+M^2)\right)}$$

$$a_{41}=\frac{\left[\varepsilon a_8 m^2 a_{28}e^{nt}+\varepsilon a_8 mxa_{28}e^{nt}+A(m+x)a_{19}\right]}{\left((m+x)^2+(m+x)-(n+M^2)\right)}\qquad a_{42}=\frac{\left[\varepsilon a_8\lambda^2 a_{29}e^{nt}+\varepsilon a_8\lambda xa_{29}e^{nt}-A(\lambda+x)a_{17}\right]}{\left((\lambda+x)^2+(\lambda+x)-(n+M^2)\right)}$$

$$a_{43}=\frac{A\eta a_{15}}{\left(\eta^2+\eta-(n+M^2)\right)}\quad a_{30}=-a_{32}+a_{33}-a_{34}+a_{35}-a_{36}+a_{37}-a_{38}+a_{39}+a_{40}-a_{41}+a_{42}+a_{43}$$

The physical quantity of most interest in such problem is the skin-friction coefficient which is defined by

$$\tau=\left(\frac{\partial U}{\partial y}\right)_{y=0}$$

From equation (3.17) we calculate τ as follows:

$$\tau=a_{11}\lambda-a_{13}q-a_{14}m+\delta\left[a_{15}\eta+a_{17}(\lambda+x)-a_{18}(q+x)-a_{19}(m+x)+a_{20}(\lambda+q)+a_{21}2q+a_{22}(m+q)\right]$$
$$\varepsilon\begin{bmatrix}a_{23}r-a_{25}x+a_{26}q-a_{27}\beta+a_{28}m-a_{29}\lambda+\\ \delta\begin{bmatrix}a_{30}s+a_{32}(r+q)-a_{33}(q+x)+a_{34}2q-a_{35}(\beta+q)+a_{36}(m+q)-a_{37}(\lambda+q)+a_{38}(r+x)\\ -a_{39}2x-a_{40}(\beta+x)+a_{41}(m+x)-a_{42}(\lambda+x)-a_{43}\eta\end{bmatrix}\end{bmatrix}e^{nt}$$

(3.18)

4.0 Discussion of results

In order to get physical insight into the problem, the effects of various physical parameters on the fluid profile were computed and represented in figures 1-11 and followed by discussion. These parameters were assigned the following values $M=1.0, G_{r\tau}=5.0, G_{rc}=1.0, \gamma=0.1, R=0.5, \varepsilon=0.1, \delta=0.1, n=0.5$

and $t=1.0$ except where stated otherwise while the values of $\Pr$, and Sc were taken to be 0.71 and 0.6 respectively for plasma. Also, $\gamma<0, \gamma=0$ and $\gamma>0$ indicate generative, no reaction and destructive chemical reaction respectively. Equation (2.13) shows that increase in α viscosity parameter leads to decrease in viscosity. Numerical values of skin friction are showed in Table 4.1. We observe that an increase in viscosity parameter, mass Grashof number or the thermal Grashof number increases skin friction whereas increase in magnetic parameter, the radiation parameter or reaction parameter leads to a decrease in skin friction coefficient.

Figures 4.1 - 4.2 depict the velocity distribution, highlighting the effect of delta and epsilon. It could be seen that increase in delta or epsilon increases velocity. In figure 4.3, we observe that generative chemical reaction leads to increase in fluid velocity while increase in destructive chemical reaction lowers the velocity. Figures 4.4 - 4.10 display the effects of $R, G_{rc}, G_{r\tau}, M, A, n$ and t on the of velocity distribution. It is observed that an increase in radiation parameter R, mass Grashof number G_{rc}, thermal Grashof number $G_{r\tau}$ or time t increases the velocity whereas an increase in magnetic number M or suction parameter A leads to a decrease in velocity distribution. Figure 4.11 shows the temperature θ profile for various values of radiation parameter. It could be seen that increase radiation parameter R reduces temperature of the fluid. Also it is noticed that a decrease in the fluid temperature with maximum value at the plate and minimum at a distance away from the plate. The effect of reaction parameter γ on the concentration of chemical species is shown in figure 4.12. We noticed that increase in reaction parameter reduces the concentration of the chemical specie

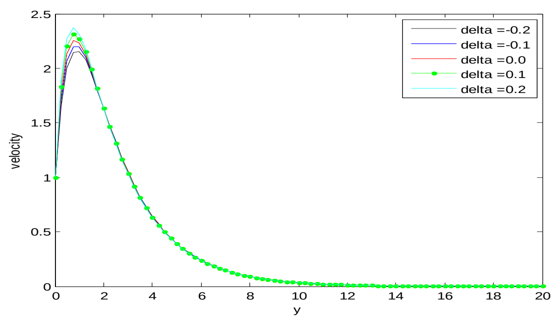

Fig 4.1: Velocity $U(y)$ distribution for various values of δ

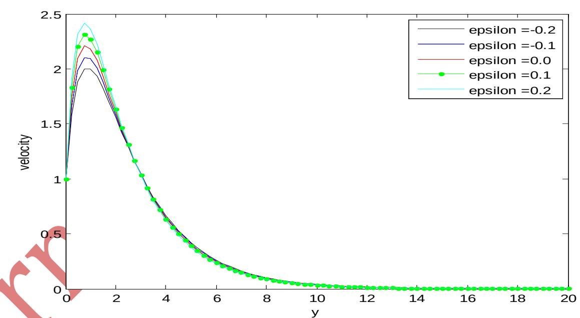

Fig 4.2: Velocity $U(y)$ distribution for various values of ε

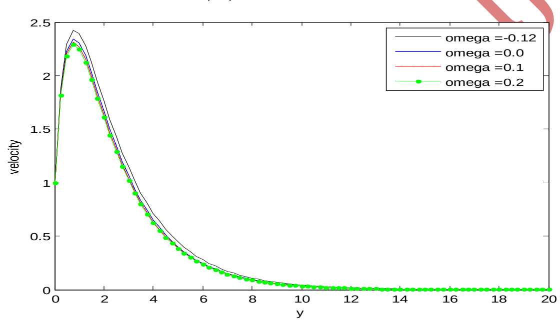

Fig 4.3: Velocity $U(y)$ distribution for various values of γ

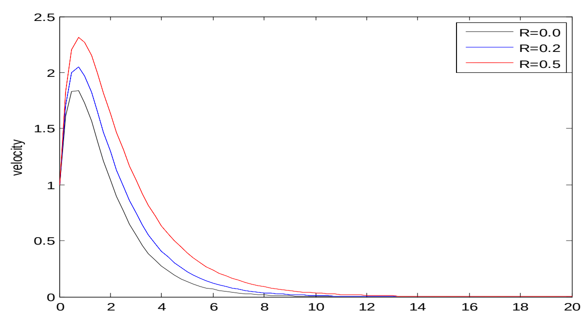

Fig 4.4: Velocity $U(y)$ distribution for various values of R

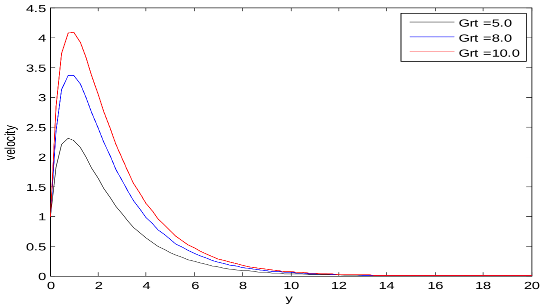

Fig 4.5: Vel. $U(y)$ distribution for various values of $G_{r\tau}$

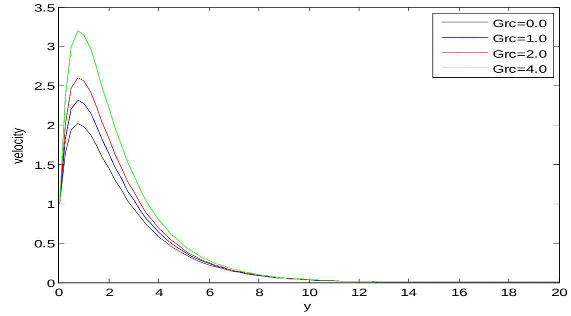

Fig 4.6: Vel. $U(y)$ distribution for various values of G_{rc}

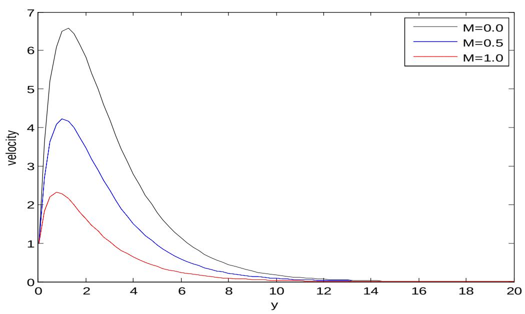

Fig 4.7: Vel. $U(y)$ distribution for various values of M

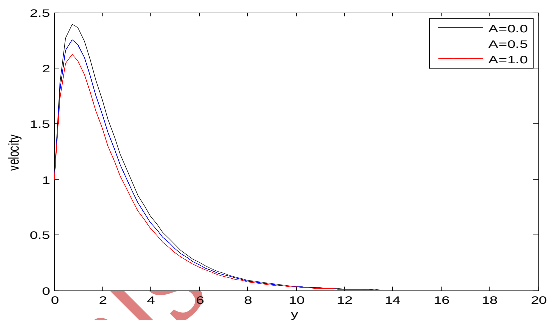

Fig 4.8: Vel. $U(y)$ distribution for various values of A

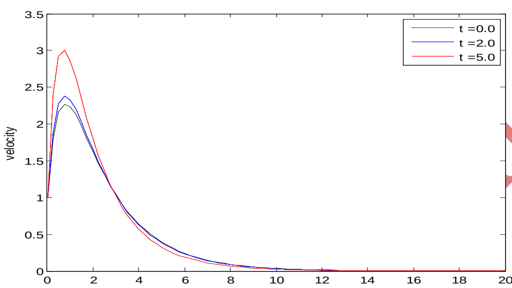

Fig 4.9: Vel. $U(y)$ distribution for various values of t

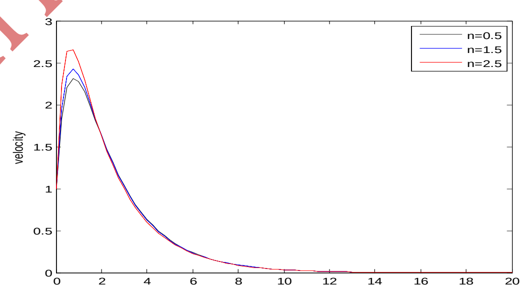

Fig 4.10: Vel. $U(y)$ distribution for various values of n

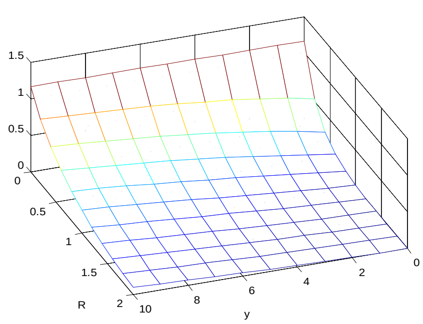

Fig 4.11: Tem. $\theta(y)$ distribution for various values of R

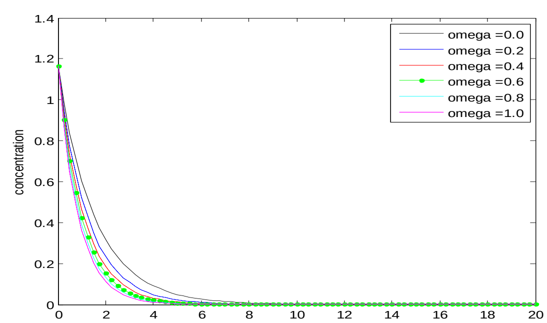

Fig 4.12: Conc $C(y)$ distribution for various values of γ

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