IJournals: International Journal of Software & Hardware Research in Engineering
ISSN-2347-4890
Volume 4 Issue 2 February, 2016

Heat and Mass Transfer of Chemically Reacting MHD Boundary Layer Flow of a Fluid with Temperature Dependent Viscosity on a Continuously Moving Vertical Plate in the Presence of Radiation and Viscous Dissipation Effects under the Influence of Constant Suction Velocity

Sogbetun L.O[1] and Fasasi Y.A.[2]

[1]Department of Computer Science, Federal College of Animal Health and Production Technology, Ibadan

[2]Department of Math and Statistics Oke-Ogun Polytechnic Saki, Ibadan

1. e-mail: mailbayo2000@yahoo.co.uk

2. e-mail:yk3fasasi@yahoo.com

Abstract

This paper examined the problem of MHD flow of a viscous fluid over a vertical porous plate with viscous dissipation in a porous medium under constant suction velocity. The fluid studied in this problem is an optically dense viscous incompressible fluid of temperature dependent viscosity. The governing equations were solved usingan asymptotic technique. Thereafter, effects of various emerging parameters on the flow fields were discussed and presented graphically.

Keywords: *MHD, vertical porous plate, viscous fluid, porous medium, suction velocity.*

1.0 Literature review

Studies abound on the problem of natural convection in nature and industrial processes. Natural convection occurs as a result of the spatial variations in density, which is caused by the non-uniform distribution of temperature or/and concentration of a dissolved substance. Numerous applications of the heat transfer by natural convection can be found in engineering applications and chemical processes. For instance, in polymer production, food processing, the cooling of an infinite metallic plate in a cooling path, glass blowing continuous casting and spinning of fibers.

In light of these applications, recently, Omowaye [1] studied a steady one-dimensional free convective flow of an incompressible electrically conducting viscous fluid on a finite plate and the temperature of the flow assumed Arrhenius dependence. He employed Nachtsheim-Swigert iteration technique along with sixth order Runge-Kutta integration scheme to solve the momentum and energy equations. Ibrahim and Makinde [2] investigated a two-dimensional, steady, viscous, incompressible, electrically conducting and laminar free convection boundary layer flow with radiation from a flat plate in a chemically reactive medium in the presence of a transverse magnetic field. The problem was solved using shooting techniques with the fourth order Runge-Kutta integration scheme. Muhammed and Shahzard [3] investigated radiation effects on MHD two dimensional stagnation point flow of a steady viscous incompressible electrically conducting fluid towards a shrinking sheet in the presence of a transverse magnetic field. The governing equations together with the associated boundary conditions are first reduced to non-linear ordinary differential equations and then solved by a method based on finite difference discretization.

All the aforementioned works on MHD flow of a viscous fluid had been confined to a case of constant viscosity. However, it is known that this physical property may change significantly with temperature, time or space variable. Gary et al [4], and Mehta and Sood [5] have established that there is wide disparity in flow characteristics of variable viscosity and that of constant. Based on this, Hosain andMunir[6] investigated a two dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate. Fang [7] analysed the influence of fluid property variation on the boundary layers of a stretching surface. Hossain et al [8] discussed the effect of radiation on free convection flow of fluid with variable viscosity from a porous plate. Mostafa [9] studied the effects of radiation and variable viscosity on hydromagnetic boundary layer flow along a continuously moving vertical plate with uniform suction and heat flux.Okedoye[10] extended Mostafa problem to include effect of chemical reaction of order one. The present study is an extension of Okedoye et al [10] to include effect of viscous dissipation term on the problem of variable viscosity on MHD flow past a porous domain in the presence of radiation and chemical reaction under suction velocity.

1.1 Nomenclature

u : Velocity along x coordinate	T' : Non dimensional fluid temperature
v : Velocity along y coordinate	C' : Non dimensional species concentration
g : Acceleration due to gravity	T : Fluid temperature
U' : Non dimensional fluid velocity	σ^* : Stefan- Boltzmann constant
T_w : Ambient temperature	B : Coefficient of mass expansion
C : Species concentration	B_τ : Coefficient of thermal expansion
C_w : Ambient species concentration	B_0 : Transverse magnetic field
ρ_w : Ambient density	σ : Electrical conductivity
ρ : Density of the fluid	τ : Skin-friction coefficient
k_t : Thermal conductivity	Sc : Schmidt number
c_p : Specific heat at constant pressure	D : Molar diffusivity
q_r : Radiative heat flux	G_{rc} : Mass grashof number
v_0 : Normal velocity at the plate	$G_{r\tau}$: Thermal grashof number
k^* : Mean absorption coefficient	δ : Delta, $0 \le \delta_o << 1$
M : Hartmann number	ε : Epsilon, $0 \le \varepsilon_o << 1$
Nu : Nuselt number	Sh : Sherwood number
ω : Angular velocity	t : Time
$\Pr$: Prandtl number	μ : Fluid viscosity

2.0 Mathematical formulation

A magnetohydrodynamic flow of viscous, incompressible, electrically conducting fluid past an infinite vertical plate in a porous medium under suction velocity is considered. The x- axis is taken along the plate in the direction of the flow and y- axis normal to it. A uniform magnetic field is applied normal to the direction of the flow. It is assumed that the magnetic Reynold number is less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. We further assumed that all the fluid properties are constant except that of the influence of density variation with temperature. Thus, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium. Initially at $t \leq 0$, the plate as well as fluid is assumed to be at the same temperature and the concentration of species is very low so that the Soret and Dofour effect are neglected [11]. When $t > 0$, the temperature of the plate is instantaneously raised (or lowered) to $T_w^{'}$ and the concentration of species is raised (or lowered) to $C_w^{'}$. Under the above assumptions and taking the usual Boussinesq's approximation into account, the governing equations for momentum, energy and concentration are presented below:

$$\frac{dv^{'}}{dy^{'}} = 0 \tag{2.1}$$

$$v^{'}\frac{dU^{'}}{dy^{'}} = \frac{1}{\rho_\infty}\frac{d}{dy^{'}}\left[\mu\frac{dU^{'}}{dy^{'}}\right] + g\beta^{*}\left(T^{'} - T_\infty\right) + g\beta\left(C^{'} - C_\infty\right) - \frac{\sigma B_0^2 U^{'}}{\rho_\infty} \tag{2.2}$$

$$v^{'}\frac{dC^{'}}{dy^{'}} = D\frac{d^2C^{'}}{dy^{'2}} - A\left(C^{'} - C_\infty\right) \tag{2.3}$$

$$v^{'}\frac{dT^{'}}{dy^{'}} = \frac{k}{\rho_\infty c_p}\frac{d^2T^{'}}{dy^{'2}} - \frac{1}{\rho_\infty c_p}\frac{dq_r}{dy^{'}} + \frac{\mu}{\rho_\infty c_p}\left(\frac{dU^{'}}{dy^{'}}\right)^2 \tag{2.4}$$

The boundary conditions:

$$U^{'} = v_0 \qquad \frac{\partial T^{'}}{\partial y^{'}} = -\frac{q}{k} \qquad C^{'} = C_w \qquad \text{at } y^{'} = 0 \tag{2.5}$$

$$U^{'} \to 0, \qquad T^{'} \to T_\infty \qquad C^{'} \to C_\infty \qquad \text{as } y^{'} \to \infty$$

From equation (2.1), we take $v^{'}\left(y^{'}\right) = -v_0$

Introducing the following non dimensional quantities:

$$y=\frac{v_0 y^{'}}{\vartheta}\quad U=\frac{U^{'}}{v_0}\quad C=\frac{C^{'}-C_\infty}{C_w-C_\infty}\quad \theta=\frac{T^{'}-T_\infty}{\left(\frac{q\vartheta}{kv_0}\right)}\quad M=\frac{B_0}{v_0}\sqrt{\frac{\sigma\vartheta}{\rho}}\quad \Pr=\frac{\mu_\infty c_p}{k_t}\quad R=\frac{16\sigma T_\infty^{\ 3}}{3k^*k}$$

$$Sc=\frac{\vartheta}{D}\quad k=\frac{k^{'}v_0^{\ 2}}{\vartheta^2}\quad \gamma=\frac{A^*\vartheta}{v_0^2}\quad E_c=\frac{v_0^{\ 2}}{c_p\left(\frac{q\vartheta}{kv_0}\right)}\quad G_{r\tau}=\frac{\vartheta g\beta^*\frac{q\vartheta}{kv_0}}{v_0^{\ 3}}\ \text{and}\ G_{rc}=\frac{\vartheta g\beta(C_w-C_\infty)}{v_0^{\ 3}}$$

Below are the non-dimensional governing equations for momentum, energy and concentration of the unsteady state and their boundary conditions:

$$\frac{d^2C}{dy^2}+S_c\frac{dC}{dy}-\gamma S_c C=0 \tag{2.6}$$

$$\frac{d^2\theta}{dy^2}+\frac{P_r}{1+R}\frac{d\theta}{dy}+\frac{P_rE_c}{1+R}\left(\frac{dU}{dy}\right)^2=0 \tag{2.7}$$

$$\frac{d}{dy}\left[\frac{\mu}{\mu_\infty}\frac{dU}{dy}\right]+\frac{dU}{dy}-M^2U=-G_{r\tau}\theta-G_{rc}C \tag{2.8}$$

The relevant boundary conditions in dimensionless form are:

$$U(y)=1\qquad C(y)=1\qquad \left.\frac{d\theta}{dy}\right|_{y=0}=-1\quad \text{at } y=0 \tag{2.9}$$

$$U(y)\to 0\qquad C(y)\to 0\qquad \theta(y)\to 0\qquad \text{as } y\to\infty$$

The fluid viscosity $\mu(\theta)$ was assumed to obey the Reynolds model [12]

$$\frac{\mu}{\mu_\infty}=e^{-\alpha\theta} \tag{2.10}$$

Where α, is a parameter depending on the nature of the fluid. Using equation (2.10) in equation (2.8), we obtain

$$\frac{d}{dy}\left[e^{-\alpha\theta}\frac{dU}{dy}\right]+\frac{dU}{dy}-M^2U=-G_{r\tau}\theta-G_{rc}C \tag{2.11}$$

3.0 Method of solution

Solving equation (2.6), we have

$$C(y) = e^{my} \qquad \text{where} \qquad m = -\frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4\gamma Sc}\right] \tag{3.1}$$

To solve equations (2.7) and (2.11), we employ asymptotic technique of the form:

$$U(y) = U_0(y) + \delta U_1(y) + \ldots + h..o.t \tag{3.2a}$$

$$\theta(y) = \theta_0(y) + \delta\theta_1(y) + \ldots + h.o.t \tag{3.2b}$$

Substituting equations (3.2) into (2.7) and (2.11) and collect order of δ , we have the following:

Corresponding to the energy equation we have

$$\frac{d^2\theta_0}{dy^2} + \frac{P_r}{1+R}\frac{d\theta_0}{dy} + \frac{P_r E_c}{1+R}\left(\frac{dU_0}{dy}\right)^2 = 0 \tag{3.3}$$

$$\left.\frac{d\theta_0}{dy}\right|_{y=0} = -1 \quad \text{at } y = 0 \qquad \theta_0(y) \to 0 \text{ as } \quad y \to \infty$$

$$\frac{d^2\theta_1}{dy^2} + \frac{P_r}{1+R}\frac{d\theta_1}{dy} + \frac{2P_r E_c}{1+R}\frac{dU_0}{dy}\frac{dU_1}{dy} = 0 \tag{3.4}$$

$$\left.\frac{d\theta_1}{dy}\right|_{y=0} = 0 \quad \text{at } y = 0 \qquad \theta_1(y) \to 0 \text{ as } \quad y \to \infty$$

Corresponding to the momentum equation we have

$$\frac{d^2U_0}{dy^2} + \frac{dU_0}{dy} - M^2U_0 = -G_{r\tau}\theta_0 - G_{rc}C \tag{3.5}$$

$$U_0(y) = 1 \text{at } y = 0 \qquad U_0(y) = 0 \text{ as } \quad y \to \infty$$

$$-\frac{d}{dy}\theta_0\frac{dU_0}{dy} + \frac{d^2U_1}{dy^2} + \frac{dU_1}{dy} - M^2U_1 = -G_{r\tau}\theta_1 \tag{3.6}$$

$$U_1(y) = 0 \quad \text{at } y \to 0 \qquad U_1(y) = 0 \quad \text{as} \quad y \to \infty$$

In order to solve equations (3.3) - (3.6), we set up asymptotic expansion of the form:

IJournals: International Journal of Software & Hardware Research in Engineering
ISSN-2347-4890
Volume 4 Issue 2 February, 2016

Let $E_c = o(\xi)$ where $\xi << 1$

$$\theta_0(y) = \theta_{00}(y) + \xi\theta_{01}(y) + \ldots + h..o.t \qquad (3.7a)$$

$$\theta_1(y) = \theta_{10}(y) + \xi\theta_{11}(y) + \ldots + h..o.t \qquad (3.7b)$$

$$U_0(y) = U_{00}(y) + \xi U_{01}(y) + \ldots + h..o.t \qquad (3.7c)$$

$$U_1(y) = U_{10}(y) + \xi U_{11}(y) + \ldots + h..o.t \qquad (3.7d)$$

Substituting equations (3.7) into (3.3)-(3.6) and collect order of ξ, we have the following:

Corresponding to the energy equation we have

$$\frac{d^2\theta_{00}}{dy^2} + \frac{P_r}{1+R}\frac{d\theta_{00}}{dy} = 0$$

$$\left.\frac{d\theta_{00}}{dy}\right|_{y=0} = -1 \text{ at } y = 0 \qquad \theta_{00}(y) \to 0 \text{ as } y \to \infty \qquad (3.8)$$

$$\frac{d^2\theta_{01}}{dy^2} + \frac{P_r}{1+R}\frac{d\theta_{01}}{dy} + \frac{P_r}{1+R}\left(\frac{dU_{00}}{dy}\right)^2 = 0$$

$$\left.\frac{d\theta_{01}}{dy}\right|_{y=0} = 0 \text{ at } y = 0 \qquad \theta_{01}(y) \to 0 \text{ as } y \to \infty \qquad (3.9)$$

$$\frac{d^2\theta_{10}}{dy^2} + \frac{P_r}{1+R}\frac{d\theta_{10}}{dy} = 0$$

$$\left.\frac{d\theta_{10}}{dy}\right|_{y=0} = 0 \text{ at } y = 0 \qquad \theta_{10}(y) \to 0 \text{ as } y \to \infty \qquad (3.10)$$

$$\frac{d^2\theta_{11}}{dy^2} + \frac{P_r}{1+R}\frac{d\theta_{11}}{dy} + \frac{2P_r}{1+R}\frac{dU_{00}}{dy}\frac{dU_{10}}{dy} = 0$$

$$\left.\frac{d\theta_{11}}{dy}\right|_{y=0} = 0 \text{ at } y = 0 \qquad \theta_{11}(y) \to 0 \text{ as } y \to \infty \qquad (3.11)$$

Corresponding to the momentum equation we have

$$\frac{d^2U_{00}}{dy^2}+\frac{dU_{00}}{dy}-M^2U_{00}=-G_{r\tau}\theta_{00}-G_{rc}C \tag{3.12}$$

$U_{00}(y)=1$ at $y=0$ $\qquad U_{00}(y)=0$ as $y\to\infty$

$$\frac{d^2U_{01}}{dy^2}+\frac{dU_{01}}{dy}-M^2U_{01}=-G_{r\tau}\theta_{01} \tag{3.13}$$

$U_{01}(y)=0$ at $y=0$ $\qquad U_{01}(y)\to 0$ as $y\to\infty$

$$-\frac{d\theta_{00}}{dy}\frac{dU_{00}}{dy}+\frac{d^2U_{10}}{dy^2}+\frac{dU_{10}}{dy}-M^2U_{10}=-G_{r\tau}\theta_{10} \tag{3.14}$$

$U_{10}(y)=0$ at $y=0$ $\qquad U_{10}(y)\to 0$ as $y\to\infty$

(3.15)

$U_{11}(y)=0$ at $y=0$ $\qquad U_{11}(y)\to 0$ as $y\to\infty$

Solving equations (3.8) –(3.15) and substitute the results into (3.7) and (3.2), we have:

$$\theta(y)=-\frac{1}{n}e^{ny}+\xi\left[a_{12}e^{\chi y}-b_6e^{2ry}-b_7e^{(r+n)y}+b_8e^{(r+m)y}-b_9e^{2ny}+b_{10}e^{(n+m)y}-b_{11}e^{2my}\right]+$$
$$\delta\xi\begin{bmatrix}a_{16}e^{qy}-b_{19}e^{(s+r)y}-b_{20}e^{(r+m+n)y}+b_{21}e^{(2r+n)y}+b_{22}e^{(2n+r)y}-b_{23}e^{(s+n)y}-b_{24}e^{(m+2n)y}+b_{25}e^{(r+2n)y}\\+b_{26}e^{3ny}+b_{27}e^{(s+m)y}+b_{28}e^{(2m+n)y}-b_{29}e^{(r+m+n)y}-b_{30}e^{(2n+m)y}\end{bmatrix} \tag{3.16}$$

$$U(y)=a_5e^{ry}+b_1e^{ny}-b_2e^{my}+\xi\left[a_{13}e^{\beta y}-b_{12}e^{\chi y}+b_{13}e^{2ry}+b_{14}e^{(r+n)y}-b_{15}e^{(r+m)y}+b_{16}e^{2ny}-b_{17}e^{(n+m)y}+b_{18}e^{2my}\right]$$
$$+\delta\left(\begin{matrix}a_9e^{sy}+b_3e^{(m+n)y}-b_4e^{(r+n)y}-b_5e^{2ny}+\\ \xi\begin{bmatrix}a_{17}e^{\eta y}+b_{31}e^{(\chi+r)y}-b_{32}e^{3ry}-b_{33}e^{(2r+n)y}+b_{34}e^{(2r+m)y}-b_{35}e^{(2n+r)y}+b_{36}e^{(n+m+r)y}-b_{37}e^{(2m+r)y}+b_{38}e^{(\chi+n)y}-\\ b_{39}e^{(2r+n)y}-b_{40}e^{(r+2n)y}+b_{41}e^{(r+m+n)y}-b_{42}e^{3ny}+b_{43}e^{(2n+m)y}-b_{44}e^{(2m+n)y}-b_{45}e^{(\chi+m)y}+b_{46}e^{(2r+m)y}+b_{47}e^{(r+n+m)}-\\ b_{48}e^{(r+2m)y}+b_{49}e^{(2n+m)y}-b_{50}e^{(n+2m)y}+b_{51}e^{3my}-b_{52}e^{(\beta+n)y}+b_{53}e^{(\chi+n)y}-b_{54}e^{(2r+n)y}-b_{55}e^{(r+2n)y}+b_{56}e^{(r+m+n)y}\\ -b_{57}e^{3ny}+b_{58}e^{(2n+m)y}-b_{59}e^{(2m+n)y}-b_{60}e^{qy}+b_{61}e^{(s+r)y}+b_{62}e^{(r+m+n)y}-b_{63}e^{(2r+n)y}-b_{64}e^{(2n+r)y}+b_{65}e^{(s+n)y}\\ +b_{66}e^{(m+2n)y}-b_{67}e^{(r+2n)y}-b_{68}e^{3ny}-b_{69}e^{(s+m)y}-b_{70}e^{(2m+n)y}+b_{71}e^{(r+m+n)y}+b_{72}e^{(2n+m)y}\end{bmatrix}\end{matrix}\right) \tag{3.17}$$

Where

$$q = -\frac{P_r}{1+R} \quad m = -\frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4\gamma Sc}\right] \quad r = -\frac{1}{2}\left[1 + \sqrt{1+4M^2}\right] \quad \eta = -\frac{1}{2}\left[1 + \sqrt{1+4M^2}\right]$$

$$n = -\frac{P_r}{1+R} \qquad s = -\frac{1}{2}\left[1 + \sqrt{1+4M^2}\right] \qquad \beta = -\frac{1}{2}\left[1 + \sqrt{1+4M^2}\right] \qquad \chi = -\frac{P_r}{1+R}$$

$$b_1 = \frac{1}{n}\frac{G_{r\tau}}{\left(n^2 + n - M^2\right)} \quad b_2 = \frac{G_{rc}}{\left(m^2 + m - M^2\right)} \quad a_5 = 1 - b_1 + b_2 \quad b_3 = \frac{mb_2}{\left((m+n)^2 + (m+n) - M^2\right)}$$

$$b_4 = \frac{a_5 r}{\left((r+n)^2 + (r+n) - M^2\right)} \quad b_5 = \frac{nb_1}{\left(4n^2 + 2n - M^2\right)} \quad a_9 = b_4 + b_5 - b_3 \quad b_6 = \frac{a_5{}^2 r^2 P_r}{4r^2(1+R) + 2rP_r}$$

$$b_7 = \frac{2a_5 r b_1 n P_r}{\left[(r+n)^2(1+R) + (r+n)P_r\right]} \quad b_8 = \frac{2a_5 r m b_2 P_r}{\left[(r+m)^2(1+R) + (r+m)P_r\right]} \quad b_9 = \frac{b_1{}^2 n^2 P_r}{\left[4n^2(1+R) + 2nP_r\right]}$$

$$b_{10} = \frac{2mnb_1 b_2 P_r}{\left[(n+m)^2(1+R) + (n+m)P_r\right]} \quad b_{11} = \frac{m^2 b_2{}^2 P_r}{\left[4m^2(1+R) + 2mP_r\right]} \quad b_{12} = \frac{G_{r\tau} a_{12}}{\left(\chi^2 + \chi - M^2\right)} \quad b_{13} = \frac{G_{r\tau} b_6}{\left(4r^2 + 2r - M^2\right)}$$

$$a_{12} = \frac{1}{\chi}\left(2rb_6 + (r+n)b_7 - (r+m)b_8 + 2nb_9 - (n+m)b_{10} + 2mb_{11}\right) \quad a_{13} = b_{12} - b_{13} - b_{14} + b_{15} - b_{16} + b_{17} - b_{18}$$

$$b_{14} = \frac{G_{r\tau} b_7}{\left((r+n)^2 + (r+n) - M^2\right)} \quad b_{15} = \frac{G_{r\tau} b_8}{\left((r+m)^2 + (r+m) - M^2\right)} \quad b_{16} = \frac{G_{r\tau} b_9}{\left(4n^2 + 2n - M^2\right)}$$

$$b_{17} = \frac{G_{r\tau} b_{10}}{\left((n+m)^2 + (n+m) - M^2\right)} \quad b_{18} = \frac{G_{r\tau} b_{11}}{\left(4m^2 + 2m - M^2\right)} \quad b_{19} = \frac{2P_r a_9 a_5 rs}{\left((s+r)^2(1+R) + (s+r)P_r\right)}$$

$$b_{20} = \frac{2P_r a_5 r(m+n)b_3}{\left((r+m+n)^2(1+R) + (r+m+n)P_r\right)} \quad b_{21} = \frac{2P_r a_5 r(r+n)b_4}{\left((2r+n)^2(1+R) + (2r+n)P_r\right)}$$

$$b_{22} = \frac{4P_r a_5 rnb_5}{\left((2n+r)^2(1+R) + (2n+r)P_r\right)} \quad b_{23} = \frac{2P_r a_9 sb_1 n}{\left((s+n)^2(1+R) + (s+n)P_r\right)}$$

$$b_{24} = \frac{2P_r b_1 nb_3(m+n)}{\left((m+2n)^2(1+R) + (m+2n)P_r\right)} \quad b_{25} = \frac{2P_r b_1 nb_4(r+n)}{\left((r+2n)^2(1+R) + (r+2n)P_r\right)}$$

$$b_{26} = \frac{4P_r b_1 b_5 n^2}{\left(9n^2(1+R) + 3nP_r\right)} \quad b_{27} = \frac{2P_r a_9 sb_2 m}{\left((s+m)^2(1+R) + (s+m)P_r\right)} \quad b_{28} = \frac{2P_r b_2 mb_3(m+n)}{\left((2m+n)^2(1+R) + (2m+n)P_r\right)}$$

$$b_{29} = \frac{2P_r b_2 mb_4(r+n)}{\left((r+m+n)^2(1+R) + (r+m+n)P_r\right)} \qquad b_{30} = \frac{4P_r b_2 mb_5 n}{\left((2n+m)^2(1+R) + (2n+m)P_r\right)}$$

$$b_{31} = \frac{a_{12}\chi a_5 r}{\left((\chi+r)^2 + (\chi+r) - M^2\right)} \qquad b_{32}\frac{b_6 a_5 2r^2}{\left(9r^2 + 3r - M^2\right)} \quad b_{33} = \frac{b_7(r+n)a_5 r}{\left((2r+n)^2 + (2r+n) - M^2\right)}$$

$$b_{34}\frac{b_8(r+m)a_5 r}{\left((2r+m)^2 + (2r+m) - M^2\right)} \qquad b_{35} = \frac{b_9 2a_5 rn}{\left((2n+r)^2 + (2n+r) - M^2\right)}$$

$$b_{36}=\frac{b_{10}a_5 r(n+m)}{\left((n+m+r)^2+(n+m+r)-M^2\right)}\qquad b_{37}=\frac{b_{11}2a_5 rm}{\left((2m+r)^2+(2m+r)-M^2\right)}$$

$$b_{38}=\frac{a_{12}b_1 n\chi}{\left((\chi+n)^2+(\chi+n)-M^2\right)}\quad b_{39}=\frac{b_6 b_1 n2r}{\left((2r+n)^2+(2r+n)-M^2\right)}$$

$$b_{40}=\frac{b_7 b_1 n(r+n)}{\left((r+2n)^2+(r+2n)-M^2\right)}\quad b_{41}=\frac{b_8 b_1 n(r+m)}{\left((r+m+n)^2+(r+m+n)-M^2\right)}\quad b_{42}=\frac{b_9 b_1 2n^2}{\left(9n^2+3n-M^2\right)}$$

$$b_{43}=\frac{b_{10}b_1 n(n+m)}{\left((2n+m)^2+(2n+m)-M^2\right)}\quad b_{44}=\frac{b_{11}b_1 n2m}{\left((2m+n)^2+(2m+n)-M^2\right)}$$

$$b_{45}=\frac{a_{12}b_2 m\chi}{\left((\chi+m)^2+(\chi+m)-M^2\right)}\qquad b_{46}=\frac{b_6 b_2 m2r}{\left((2r+m)^2+(2r+m)-M^2\right)}$$

$$b_{47}=\frac{b_7 b_2 m(r+n)}{\left((r+n+m)^2+(r+n+m)-M^2\right)}\quad b_{48}=\frac{b_8 b_2 m(r+m)}{\left((r+2m)^2+(r+2m)-M^2\right)}$$

$$b_{49}=\frac{b_9 b_2 m2n}{\left((2n+m)^2+(2n+m)-M^2\right)}\qquad b_{50}=\frac{b_{10}b_2 m(n+m)}{\left((n+2m)^2+(n+2m)-M^2\right)}$$

$$b_{51}=\frac{b_{11}b_2 2m^2}{\left(9m^2+3m-M^2\right)}\quad b_{52}=\frac{a_{13}\beta}{\left((\beta+n)^2+(\beta+n)-M^2\right)}\quad b_{53}=\frac{b_{12}\chi}{\left((\chi+n)^2+(\chi+n)-M^2\right)}$$

$$b_{54}=\frac{b_{13}2r}{\left((2r+n)^2+(2r+n)-M^2\right)}\qquad b_{55}=\frac{b_{14}(r+n)}{\left((r+2n)^2+(r+2n)-M^2\right)}$$

$$b_{56}=\frac{b_{15}(r+m)}{\left((r+m+n)^2+(r+m+n)-M^2\right)}\quad b_{57}=\frac{b_{16}2n}{\left(9n^2+3n-M^2\right)}\quad b_{58}=\frac{b_{17}(n+m)}{\left((2n+m)^2+(2n+m)-M^2\right)}$$

$$b_{59}=\frac{b_{18}2m}{\left((2m+n)^2+(2m+n)-M^2\right)}\quad b_{60}=\frac{G_{r\tau}a_{16}}{\left(q^2+q-M^2\right)}\quad b_{61}=\frac{G_{r\tau}b_{19}}{\left((s+r)^2+(s+r)-M^2\right)}$$

$$b_{62}=\frac{G_{r\tau}b_{20}}{\left((r+m+n)^2+(r+m+n)-M^2\right)}\quad b_{63}=\frac{G_{r\tau}b_{21}}{\left((2r+n)^2+(2r+n)-M^2\right)}$$

$$b_{64}=\frac{G_{r\tau}b_{22}}{\left((2n+r)^2+(2n+r)-M^2\right)}\quad b_{65}=\frac{G_{r\tau}b_{23}}{\left((s+n)^2+(s+n)-M^2\right)}$$

$$b_{66}=\frac{G_{r\tau}b_{24}}{\left((m+2n)^2+(m+2n)-M^2\right)}\quad b_{67}=\frac{G_{r\tau}b_{25}}{\left((r+2n)^2+(r+2n)-M^2\right)}\qquad b_{68}=\frac{G_{r\tau}b_{26}}{\left(9n^2+3n-M^2\right)}$$

$$b_{69}=\frac{G_{r\tau}b_{27}}{\left((s+m)^2+(s+m)-M^2\right)}\qquad b_{70}=\frac{G_{r\tau}b_{28}}{\left((2m+n)^2+(2m+n)-M^2\right)}$$

$$b_{71}=\frac{G_{r\tau}b_{29}}{\left((r+m+n)^2+(r+m+n)-M^2\right)}\qquad b_{72}=\frac{G_{r\tau}b_{30}}{\left((2n+m)^2+(2n+m)-M^2\right)}$$

$$a_{16}=\frac{1}{q}\begin{bmatrix}(s+r)b_{19}+(r+m+n)b_{20}-(2r+n)b_{21}-(2n+r)b_{22}+(s+n)b_{23}+(m+2n)b_{24}-\\(r+2n)b_{25}-3nb_{26}-(s+m)b_{27}-(2m+n)b_{28}+(r+m+n)b_{29}+(2n+m)b_{30}\end{bmatrix}$$

$$a_{17} = -b_{31} + b_{32} + b_{33} - b_{34} + b_{35} - b_{36} + b_{37} - b_{38} + b_{39} + b_{40} - b_{41} + b_{42} - b_{43} + b_{44} + b_{45} - b_{46} - b_{47} + b_{48} - b_{49} + b_{50} - b_{51} + b_{52} - b_{53} + b_{54} + b_{55} - b_{56} + b_{57} - b_{58} + b_{59} + b_{60} - b_{61} - b_{62} + b_{63} + b_{64} - b_{65} - b_{66} + b_{67} + b_{68} + b_{69} + b_{70} - b_{71} - b_{72}$$

$$a_{43} = \frac{A\eta a_{15}}{\left(\eta^2 + \eta - \left(n + M^2\right)\right)}$$

$$a_{30} = -a_{32} + a_{33} - a_{34} + a_{35} - a_{36} + a_{37} - a_{38} + a_{39} + a_{40} - a_{41} + a_{42} + a_{43}$$

The physical quantity of most interest in such problem is the skin-friction coefficient which is defined by $\tau = \left(\frac{\partial U}{\partial y}\right)_{y=0}$

From equation (3.17) we calculate τ as follows:

$$\tau = a_5 r + b_1 n - b_2 m + \xi\left[a_{13}\beta - b_{12}\chi + b_{13}2r + b_{14}(r+n) - b_{15}(r+m) + b_{16}2n - b_{17}(n+m) + b_{18}2m\right]$$

$$+\delta\left(\begin{array}{l} a_9 s + b_3(m+n) - b_4(r+n) - b_5 2n + \\ \xi\left[\begin{array}{l} a_{17}\eta + b_{31}(\chi + r) - b_{32}3r - b_{33}(2r+n) + b_{34}(2r+m) - b_{35}(2n+r) + b_{36}(n+m+r) - b_{37}(2m+r) + b_{38}(\chi+n) - \\ b_{39}(2r+n) - b_{40}(r+2n) + b_{41}(r+m+n) - b_{42}3n + b_{43}(2n+m) - b_{44}(2m+n) - b_{45}(\chi+m) + b_{46}(2r+m) \\ + b_{47}(r+n+m) - b_{48}(r+2m) + b_{49}(2n+m) - b_{50}(n+2m) + b_{51}3m - b_{52}(\beta+n) + b_{53}(\chi+n) - b_{54}(2r+n) - b_{55}(r+ \\ + b_{56}(r+m+n) - b_{57}3n + b_{58}(2n+m) - b_{59}(2m+n) - b_{60}q + b_{61}(s+r) + b_{62}(r+m+n) - b_{63}(2r+n) - b_{64}(2n+r) \\ + b_{65}(s+n) + b_{66}(m+2n) - b_{67}(r+2n) - b_{68}3n - b_{69}(s+m) - b_{70}(2m+n) + b_{71}(r+m+n) + b_{72}(2n+m) \end{array}\right. \end{array}\right.$$

4.0 Discussion of Results

Some numerical calculations were carried out for the non-dimensional velocity U, temperature θ, concentration C and skin friction τ based on various values of the varying parameters $M, G_{r\tau}, G_{rc}, \gamma, R, \varepsilon, \delta, n,$ and t with fixed values for $\Pr$, and Sc. These parameters were assigned the following values $M = 1.0, G_{r\tau} = 5.0, G_{rc} = 1.0, \gamma = 0.1, R = 0.5, \varepsilon = 0.1, \delta = 0.1, n = 0.5$

and $t = 1.0$ except where stated otherwise while the values of $\Pr$, and Sc were taken to be 0.71 and 0.6 respectively for plasma. Also, $\gamma < 0, \gamma = 0$ and $\gamma > 0$ indicate generative, no reaction and destructive chemical reaction respectively. Equation (2.13) shows that increase in α viscosity parameter leads to decrease in viscosity.

Figures 4.1 - 4.2 depict the velocity distribution, highlighting the effect of delta and xi. It could be seen that increase in delta or xi increases velocity. In figure 4.3, we observe that generative chemical reaction leads to increase in fluid velocity while increase in destructive chemical reaction lowers the velocity. Figures 4.4 - 4.7 display the effects of $R, G_{rc}, G_{r\tau}$ and M on the of velocity distribution. It is observed that an increase in radiation parameter R, mass Grashof number G_{rc}, thermal Grashof number or $G_{r\tau}$ increases the velocity whereas an increase in magnetic number M leads to a decrease in velocity distribution. Figure 4.8 shows the temperature θ profile for various values of radiation parameter. It could be seen that increase radiation parameter R reduces temperature of the fluid. Also it is noticed that a decrease in the fluid temperature with maximum value at the plate and minimum at a distance away from the plate. The effect of reaction parameter γ on the concentration of chemical species is shown in figure 4.9. We noticed that increase in reaction parameter reduces the concentration of the chemical specie.

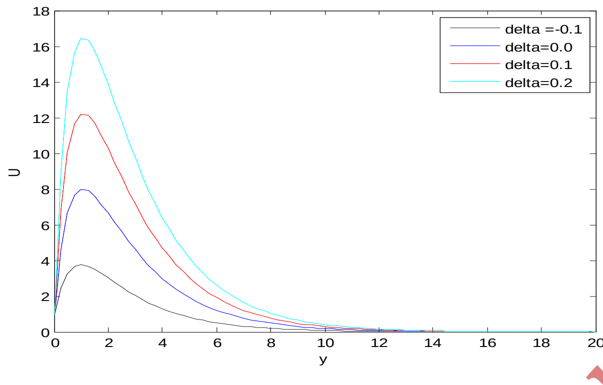

Fig 4.1: velocity $U(y)$ **distribution for various values of** δ

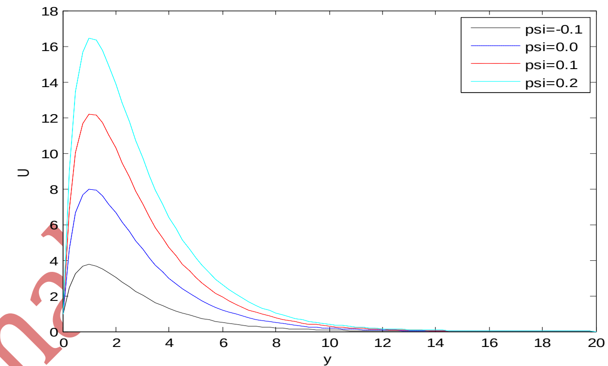

Fig 4.2: velocity $U(y)$ **distribution for various values of** ξ

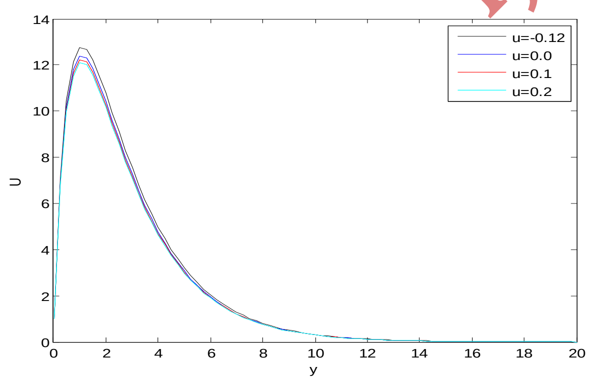

Fig 4.3: velocity $U(y)$ **distribution for various values of** γ

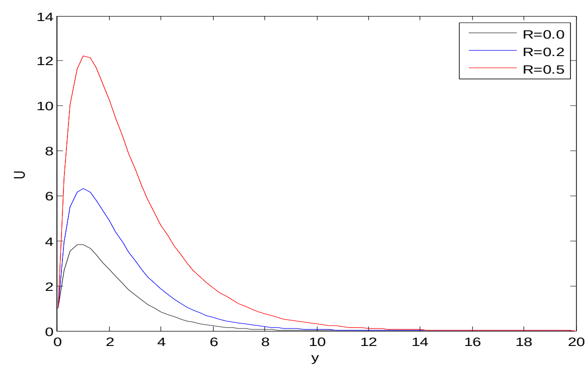

Fig 4.4: velocity $U(y)$ **distribution for various values of** R

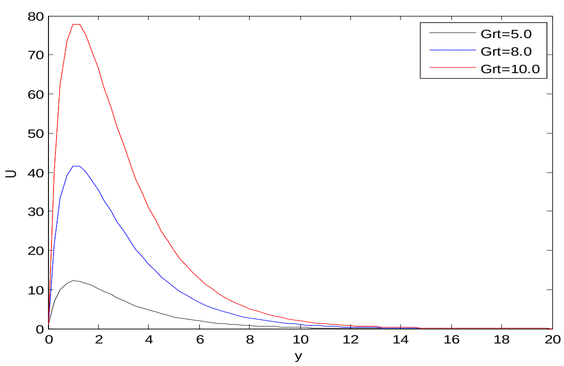

Fig 4.5: velocity $U(y)$ **distribution for various values of** Grt

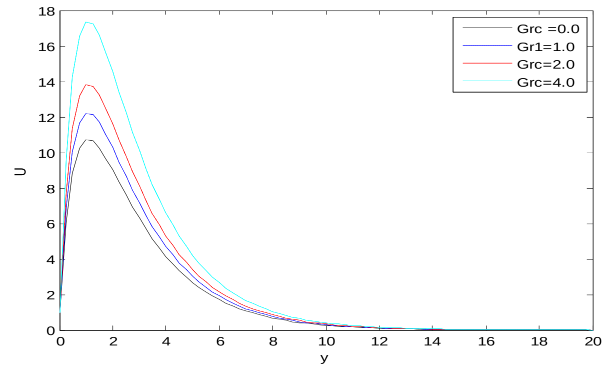

Fig 4.6: velocity $U(y)$ **distribution for various values of** Grc

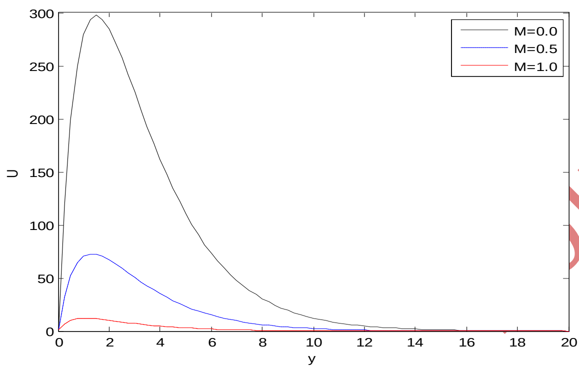

Fig 4.7: velocity $U(y)$ **distribution for various values of** M

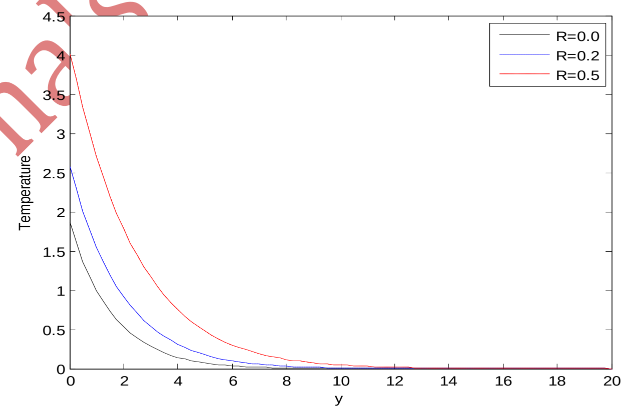

Fig 4.8: temperature $\theta(y)$ **distribution for various values of** R

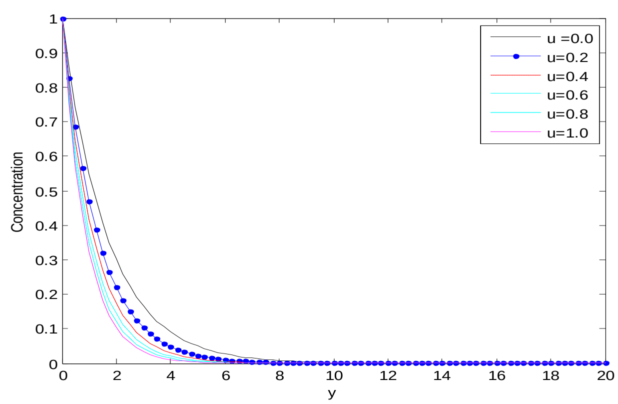

Fig 4.9: concentration $C(y)$ **distribution for various values of** γ

REFERENCE

[1] Omowaye, A.J., On Steady Free Convective Hydro Magnetic Flow of a Reactive Viscous Fluid in a Bounded Domain, *The Pacific Journal of Science and Technology*, vol.**12**. No. 1, pp.180-187,2011.

[2] Ibrahim, S. Y. and Makinde, O. D., Radiation Effect on Chemical Reacting Magnetohydrodynamic (MHD) Boundary Layer Flow of Heat and Mass Transfer through a Porous Vertical Flat Plate, *Int. J. of the Phy. Scie.*vol.**6(6)**, pp.1508-1516, 2011.

[3] Muhammed, A. and Shahzad, A., Radiation Effects on MHD Boundary Layer Stagnation Point Flow Towards a Heated Shrinking Sheet, *World Applied Sciences Journal* vol.**13(7)**: pp.1748-1756, 2011.

[4] Gary, J., Kassory, D. R., Tadjeran, H., and Zebib, A., The effect of significant viscosity variation on convective heat transport in water-saturated porous media, *J. Fluid Mech.*vol.pp.117, 233-249, 1982.

[5] Mehta K.N., Sood S., Transient free convective flow with temperature dependent viscosity in a fluid saturated porous medium, *Int. J. Engrg. Sci.*vol. 30, pp.1083 -1087, 1995.

[6] Md. Anwar Hossain and Md. S. Munir, Mixed Convection Flow from a Vertical Plate with Temperature Dependent Viscosity, *Int. J. Therm. Sci.*, vol. **39,** pp.173-183, 2000.

[7] T. Fang, Influence of Fluid Property Variation on the Boundary Layers of a Stretching Surface,*ActaMechanica.* vol. **171,** pp.105-118, 2004.

[8] M. Anwar Hossain, K. Khanafer and K. Vafai, The Effect of Radiation on Free Convection Flow of Fluid with Variable Viscosity from a Porous Vertical Plate, *Int. J. Therm. Sci.*, vol. **40,** pp.115-124, 2001.

[9] Mostafa, A. A. M., Variable Viscosity Effects on Hydromagnetic Boundary Layer Flow along a Continuously Moving Vertical Plate in the Presence of Radiation. *Applied Mathematical Sciences,* vol. **1**, No.17, pp.799-814, 2007.

[10]. Okedoye,A. M., and Frayola,P. I., Effects of Variable Viscosity on MHD Boundary Layer Flow on a Continuously Moving Vertical Plate in the Presence of Radiation and a Chemical Reaction of Order 1, *Journal of the Nigerian Association of Mathematical Physics (NAMP)*, vol. **13,**pp. 201-210, 2008.

[11] Gebhart, B., and Pera, L., The Nature of Vertical Natural Convection Flow Resulting from -the Combined Bouyancy Effects Thermal and Mass Diffusion, *Int. J. of Heat Mass Transfer*, vol.**14**, pp.2025 – 2050, 1971.

[12]. Massoudi, M. and Phuoc, T. X., Flow of a Generalized Second Grade Non-Nowtonian Fluid with Variable Viscosity, *Continuum Mech. Thermodyn.*, vol.**16**, pp.529-538, 2004.