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FIXED POINTS FOR WEAK COMPATIBLE AND SEMICOMPATIBLE MAPPINGS IN FUZZY METRIC SPACE

Dr. NidhiAsthana
Department of Engineering Mathematics
STME,SVKM'S-NMIMS (Deemed-to-be-University), Indore Campus,
Indore,INDIA
drnidhi80@gmail.com

Abstract: *The aim of this research article is to establish a common fixed point theorem and find some fixed points in fuzzy metric spaces with the concept of weak and semi compatible mappings. The main result of our research paperenlarges previous ones in fuzzy metric spaces.*

Keywords: *Fuzzy Metric space, common fixed point, weak compatible mappings, semi compatible mappings*

I. Introduction

In 1965 Zadeh [19] proposed a mathematical approach by describing fuzzy set. In the subsequent decade Kramosil and Michalek [10] offered the notion of fuzzy metric spaces in 1975, which revealed a way for further development of analysis and topology in such spaces. The outstanding feature of fuzzy set is that it allows partial membership for elements in its domain, while in ordinary set theory particular element has either full membership or no membership, no intermediate situation is considered. Scholars are using fuzzy centeredphilosophies to rise the effectiveness and to overcome the disadvantages of ordinary sets. To use the fuzzy notion in analysis and topology, several researchers have well-defined fuzzy metric space in various ways [5,6,9,11].In recent times lots of work has been done on applying fuzziness Tripathy and Borgogain [15] Tripathy and Dutta([16], [17]), Tripathy and Debnath [18] and others. In the section 3 we improve the result of [1] and [2]. For the sake of comprehensiveness, we bring into being with some definitions and introductoryconceptions [1, 3].

II. Definitions and Preliminaries

Definition 2.1 Let(X, d) be any merit linear space. A fuzzy set in X is a function with domain X and values in [0, 1].

Definition 2.2 If A is a fuzzy set and x € X, the function values A(x) (or $\mu_A(x)$) is called the grade of membership of x in A. The collection of all fuzzy sets in X is denoted by F(x).

Definition 2.3 Let X is an arbitrary set and Y be any metric linear space. Then F is called a fuzzy mapping if and only it F is a mapping from the set X into Y.

Definition 2.4 A Point p € X is called a fixed point of a fuzzy mapping F if $F_p(p) \geq Fp(x), \forall x \in X$.

Lemma 2.1 [3] $F_p(p) \geq F_p(x) \leftrightarrow p \in \hat{F}(p), \forall x \in X$.

Definition 2.5 [12] A binary operation *: [0,1] ×[0,1]→[0,1] is continuous t-norm if ([0,1],*) is a topological monoid with unit 1 such that $a*b \leq c*d$, whenever $a \leq c$ and $b \leq d$, for all a, b, c, d € [0,1].

Definition 2.6 [7] The 3 tuple (X,M,*) is said to be a fuzzy metric space if X is an arbitrary set, * a continuous t-norm and M, a fuzzy set on $x^2 \times (0,\infty)$ satisfying the following conditions:

(F-1) $M(x,y,t) > 0$;
(F-2) $M(x,y,t) = 1$, if and only if $x = y$;
(F-3) $M(x,y,t) = M(y,x,t)$;
(F-4) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$;
(F-5) $M(x,y,.) : (0,\infty) \to [0,1]$ is continuous for all $x, y, z \in X$ and $s > 0$.

Definition 2.7 A sequence $\{x_n\}$ in fuzzy metric space (X, M, *) converges to x if and only if $M(x_n, x, t) \to 1$, as $n \to \infty$, for all $t > 0$.

Definition 2.8 A sequence $\{x_n\}$ in fuzzy metric space (X, M, *) is said to be Cauchy sequence if and only if $M(x_n, x_{n+p}, t) \to 1$, as $n \to \infty$, for all $t > 0$ and $p > 0$.

Definition 2.9 [19] Fuzzy metric space (X, M, *) is said to be complete if every Cauchy sequence in (X , M, *) is convergent sequence.

Lemma 2.2 [4] Let a sequence $\{y_n\}$ in fuzzy metric space (X, M, *) with the condition $\lim_{t\to\infty} M(x,y,t) = 1$ $\forall$ x,y ϵ X. If there exist a number $k\epsilon(0,1)$ such that $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$, $\forall$ $t > 0$. Then $\{y_n\}$ is a Cauchy sequence in X.

Definition 2.10 [14] A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the mappings (A,S) are said to be weak compatible if Ax =Sx implies that ASx = SAx

Definition 2.11 [8] Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the mappings are said to be compatible if

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, \ \forall\ t > 0$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$$

Proposition 2.1 [13] Self mappings A and S of a fuzzy metric space (X, M, *) are compatible, then they are weak compatible.

Definition 2.12 Let A and S be mappings from a fuzzy metric space (X, M,*) into itself. Then the mappings are said to be semi-compatible if

$$\lim_{n\to\infty} M(ASx_n, Sx, t) = 1, \ \forall\ t > 0$$

Whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X.$$

It follows that if (A,S) is semi- compatible and Ay = Sy, then ASy = SAy. Thus if the pair (A,S) is semi-compatible, then it is weak compatible.

III. Main Result

In this section we establish the main result of this paper.

Theorem 3.1 Let A,B,J,R,S and T be self- mapping of complete fuzzy metric space (X,M,*) satisfying the

$AB(X) \subseteq T(X)$, $JR(X)) \subseteq S(X)$ (3.1)

The pair (AB.S) is semi-compatible and (JR,T)is weak compatible (3.2)

One of AB or S is continuous; (3.3)

There exists $k \in (0,1)$ such that for all $x,y \in X$ and $t > 0$,

$M(ABx, JRy, kt) \geq \text{Min}\{M(Sx,Ty,t), M(ABx,Sx,t), M(JRy, Ty, t), M(JRy,Sx,t), M(ABx, Ty,t)\}$ (3.4)

Then AB, JR, S and T have unique common fixed point in X.

Proof: Let $x_0 \in X$ be any arbitrary point as AB (X) $\subseteq$ T (X) and JR (X) $\subseteq$ S (X) , there exist x_1 , $x_2 \in X$ such that $ABx_0 = Tx_1$, and $JR_{x1} = S_{x2}$.
Inductively, construct sequence $\{y_n\}$ and $\{x_n\}$ in X such that

$$y_{2n+1}, = ABx_{2n} = Tx_{2n+1} , JRx_{2n+1} = SX_{2n+2} = y_{2n+2}, \text{ for } n = 0, 1, 2,\dots .$$

Now using (3.4) with $x=x_{2n}$, $y=x_{2n+1}$, we get:
$M(y_{2n+1}, y_{2n+2},kt)=M(ABx_{2n}, JRx_{2n+1},kt)$
$\geq Min\{M(Sx_{2n}, Tx_{2n+1},t), M(ABx_{2n}, Sx_{2n}, t), M(JRx_{2n+1}, Tx_{2n+1}, kt), M(JRx_{2n+1},Tx_{2n+1},t), M(Abx_{2n}, Tx_{2n+1},t))\}$
$\geq Min\{M(y_{2n},y_{2n+1},t),M(y_{2n+1} ,y_{2n+2}, t), 1\}$
$\geq M(y_{2n},y_{2n+1},t)$
Thus, for any n and t, we have $M (y_n,y_{n+1},kt) \geq M (y_{n-1}, y_n, t)$. (3.5)

Hence by lemma 2.2 $\{y_n\}$ is a Cauchy sequence in X, which is complete. Therefore $\{y_n\}$ and its subsequences, converge to u $\in$ X, that is

$\{ABx_{2n}\} \to u$, $\{JRx_{2n+1}\} \to u$ (3.6)

$\{Sx_{2n}\} \to u$, $\{Tx_{2n+1}\} \to u$. (3.7)

Case I : (S is continuous). In this case, we have $S(ABx_{2n}) \to SU$, $S^2x_{2n} \to S_U$. (3.8)
The semi-compatibility of the pair (AB, S) gives $\lim_{n\to\infty}(AB)(Sx_{2n}) = Su$. (3.9)

Step 1: By putting $x = Sx_{2n}$, $y = x_{2n+1}$ in (3.4),
$M((AB)Sx_{2n}, (JR)x_{2n+1}, kt) \geq Min\{ M (Sx_{2n}, Tx_{2n+1}, t) , M((AB)Sx_{2n}, SSx_{2n}, t), M(JRx_{2n+1} , Tx_{2n+1},kt) M(JRx_{2n+1}, SSx_{2n},t), M((AB)Sx_{2n}, Tx_{2n+1},t)\}$
Letting $n \to \infty$, using (3.6),(3.7),(3.8),(3.9)
$M(Su ,u,kt) \geq Min\{ M (Su ,u,t), M(Su ,Su,t), M(u ,u,t) M(u,Su,t) M(Su,u,t)\} \geq Min\{M(Su,u,t),1\} \geq M(Su,u,t)$
Hence Su = u. (3.10)

Step 2: By putting $x = u$, $y = x_{2n+1}$ in (3.4), we obtain that
$M((AB)u, (JR)x_{2n+1}, kt) \geq Min\{ M (Su, Tx_{2n+1}, t) , M((AB)u, Su, t), M(JRx_{2n+1} , Tx_{2n+1},kt) M(JRx_{2n+1}, Su,t), M((AB)u, Tx_{2n+1},t)\}$
Taking limit as $n \to \infty$ and using (3.7), (3.10), we get.
$M(ABu, u, kt) \geq Min\{ M (u, u, t) , M((AB)u, u, t), M(u,u,t) M(u,u,t), M((AB)u,u,t)\} \geq Min\{M((AB)u,u,t),1\}$
$\geq M((AB)u,u,t)\}$
Thus u=ABu, Hence, ABu = u = Su. (3.11)

Step 3: As AB(X) $\subseteq$ T(X), there exists w $\in$ X such that ABu=Su=u Tw. By putting $x=x_{2n}$, y= w in (3.4) we obtain that:
$M((AB)x_{2n}, (JR)v, kt) \geq Min\{ M (Sx_{2n}, Tv, t) , M((AB)x_{2n}, Sx_{2n}, t), M(JRv , Tv,kt), M(JRv, Sx_{2n},t), M((AB)x_{2n}, Tv,t)\}$
Taking limit as $n \to \infty$ and using (3.6) and (3.7), we get
$M(u, JRv, kt) \geq Min\{ M (u, u, t) , M(u, u, t), M(JRv,u,t) M(JRv,u,t), M(u,u,t)\} \geq Min\{M(JRv,u,t),1\}$
$\geq M(JRv,u,t)\}$
Thus u = JRv. Therefore JRv = Tv = u. since (JR,T) is weak compatible, we get that T(JRv)=(JR)Tv,
That is, JRu=Tu. (3.12)

Step 4: By putting x=u, y=u in condition (3.4) and using (3.11) and (3.12), we obtain that
$M((AB)u, (JR)u\, kt) \geq Min\{ M (Su, Tu, t) , M((AB)u, Su, t), M(JRu , Txu,kt) M(JRu, Su,t), M((AB)u, Tu,t)\}$
Using 3.10 and 3.11 we get
$M(u, JRu, kt) \geq Min\{ M (u, JRu, t) , M(u, u, t), M(JRu,JRu,t) M(JRu,u,t), M(u,u,t)\} \geq Min\{M(JRu,u,t),1\}$
$\geq M(JRu,u,t)\}$
Thus JRu=u.

Therefore u=ABu=Su=JRu=Tu, that is, u is a common fixed point of AB, JR, S and T.

Case II (AB is continuous). In this case, we have $(AB)Sx_{2n} \to ABu$.

The semi-compatibility of the pair (AB,S) gives $(AB)Sx_{2n} \to Su$.

By uniqueness of limit in fuzzy metric space, we obtain that Abu = Su.

Step 5: By putting x = u $y=x_{2n+1}$ in (3.4) ,we get

$M((AB)u, (JR)x_{2n+1}, kt) \geq Min\{ M (Su, Tx_{2n+1}, t) , M((AB)u, Su, t), M(JRx_{2n+1} , Tx_{2n+1,}kt)\ M(JRx_{2n+1}, Su,t), M((AB)u, Tx_{2n+1},t)\}$

Taking limit as $n \to \infty$ and using (3.5), (3.6) & (3.10),we get

$M(ABu, u, kt) \geq Min\{ M (u, u, t) , M((AB)u, u, t), M(u,u,t)\ M(u,u,t), M((AB)u,u,t)\} \geq Min\{M((AB)u,u,t),1\} \geq M((AB)u,u,t)\}$

Thus u=Abu and rest of the proof follows from step 3 onwards of the pervious case.

Step 6. Uniqueness: Let z be another common fixed point of AB, JR, S and T. Then z = ABz = JRz = Sz = Tz.

Putting x = u and y = z in (3.4), We get

$M((AB)u, (JR)z, kt) \geq Min\{ M (Su, Tz, t) , M((AB)u, Su, t), M(JRz , Tz_{,}kt)\ M(JRz, Su,t), M((AB)u, Tz,t)\}$

$M(u, z, kt) \geq Min\{ M (u, z\ t) , M(u, u, t), M(z,z,t)\ M(z,u,t), M(u,u,t)\} \geq Min\{M(z,u,t),1\} \geq M(z,u,t)\}$

That is u = z. Therefore u is the unique common fixed point of the self-mappings AB, JR, S and T.

Corollary 3.1 Let A, B, S, T, J and R be self-mapping of a complete fuzzy metric shape (X, M, *) satisfying (3.1), (3.4), (3.5) and that the pairs (AB, S) and (JR, T) are semi compatible, One of AB, JR, S or T is continuous. Then AB, JR, S and T have a unique common fixed point in X.

Proof: As semi-compatibility implies weak compatibility the proof follows from theorem 3.1

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