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QR-Submanifold of a Manifold with a Kenmotsu 3-Structure

Author: Dr. Gajendra Singh

Affiliation: University Department of Mathematics, Nilamber-Pitamber University, Medininagar, Jharkhand (India) - 822102

E-mail: drgajendrasingh2@gmail.com

ABSTRACT

A. Bejancu (1986) introduced the QR-Submanifold of a Quaternion Kaehler manifold which is based on the action of local basis $\{\phi_1, \phi_2, \phi_3\}$ on each tangent space of the manifolds where $\phi_1,\ \phi_2, \phi_3$ are almost Hermitian structures.

In this paper we have taken these structure as a Kenmotsu 3-Structure and studied the properties of QR-Submanifold which includes the integrability of the distribution $D \ and \ D^{\perp}$.

Kewords : Kenmotsu 3-Structure, QR-Submanifold

1. INTRODUCTION

A(4m + 3) dimensional differentiable $\widetilde{M}$ is said to have an almost contact 3-Structure if there exists three almost contact structures $(\phi_a, \xi_a, \eta_a), a = 1, 2, 3$) satisfying the following relation:

$$\eta_a(\xi_b) = \eta_b(\xi_a) \qquad (1.1)$$

$$\phi_a{}^2 = -I + \eta_a \otimes \xi_a \qquad (1.2)$$

$$\phi_a \xi_b = -\phi_b \xi_a = \xi_c \qquad (1.3)$$

$$\eta_a \, 0 \, \phi_b = -\eta_b \, 0 \phi_a = \eta_c \qquad (1.4)$$

$$\phi_a \, 0 \, \phi_b - \xi_b \otimes \eta_a = -\phi_b \, 0 \, \phi_a + \xi_a \otimes \eta_b = \phi_c \qquad (1.5)$$

for each cyclic permutation (a, b, c) of (1, 2, 3). Let g be a Riemannian metric on $\widetilde{M}$ satisfying.

$$\eta_a(X) = g(X, \xi_a) \qquad (1.6)$$

$$g(\phi_a X, \phi_a Y) = g(X, Y) - \eta_a(X)\eta_a(Y) \qquad (1.7)$$

for any X,Y tangent to $\widetilde{M}$ and a = 1, 2, 3. $\widetilde{M}$ is called manifold with a Kenmotsu 3-Structure if each $(\phi_a, \xi_a, \eta_a, g)(a = 1, 2, 3)$ i.e.

$$(\widetilde{\nabla}_X \phi_a)Y = g(\phi_a X, Y)\xi_a - \eta_a(Y)\phi_a X \qquad (1.8)$$

$$\widetilde{\nabla}_X \xi_a = X - \eta_a(X)\xi_a = -\phi_a{}^2 X \qquad (1.9)$$

Where $\widetilde{\nabla}$ is Levi Cirita connection with respect to metric g on $\widetilde{M}$.

2. DEFINITION

A Submanifold M of a manifold $\widetilde{M}$ with an almost contact three structure is said to be a quaternion real Sub manifold (QR-Submanifold) if there exists a vector Sub-bundle υ of the normal bundle such that we have. (Bejancu 1986)

$$\phi_a(\upsilon_x) = \upsilon_x$$

and $\phi_a(\upsilon_x{}^{\perp}) \subset T_x M$ for each $X \in M,\ a = 1, 2, 3$ where $\upsilon^{\perp}$ is the complementary orthogonal Sub-bundle to υ in $TM^{\perp}$

Then we denote $D_{ax} = \phi_a \upsilon_x$ such that $D_1 x, D_2 x$ and $D_3 x$ are mutually orthogonal sub-spaces of $T_x M$.

We take $D_x{}^{\perp} = D_{1x} \oplus D_{2x} \oplus D_{3x}$ such that $D^{\perp} : x \longrightarrow D_x{}^{\perp}$is a 3s-dimensional distribution defined on M, where $s = \dim D_x{}^{\perp}$, we also have

$\phi_a D_a X = \upsilon_x{}^{\perp}, \phi_a(D_{bx}) = D_{cx}$ for each $X \in M,\ a = 1,2,3$

Where a, b and c is a center permutation of (1, 2, 3). We denote by D the complementary orthogonal distribution to $D^{\perp}$ in TM such that $\phi_a(D_x) = D_x$ for $x \in M, a = 1, 2, 3$. The Gauss and Weingarten formula are respectively given by,

$$\widetilde{\nabla}_x Y = -\nabla_X Y + h(X, Y) \qquad (1.10)$$

$$\widetilde{\nabla}_x V = -A_V X + \nabla_X{}^{\perp} V \qquad (1.11)$$

Where $V \in \Gamma(TM)$ and $X, Y \in \Gamma(TM)$, ∇ is the connection on M with respect to the induced metric on $M, \nabla^{\perp}$ is the connection in the normal bundle $TM^{\perp}$ h and A are second fundamental forms related by

$$g(h(X, Y), V) = g(A_V X, Y) \qquad (1.12)$$

we put $\phi_a V = B_a V + C_a V$ (1.13)

3. INTEGRABILITY OF DISTRIBUTION AND SOME BASIC RESULTS

Let M be a QR-Submanifold of a manifold with a Kenmotsu 3-Structure. We denote by P the projection of TM to D and take a local vector field of orthogonal frames $\{V_1, \dots\dots, V_s\}$ on the vector sub-bundle $\upsilon^{\perp}$ and $TM^{\perp}$. Then on the distribution $D^{\perp}$ we have local vector field orthonormal frames. (Bejancu, A. 1986)

$$\{E_{11}, \dots\dots\dots\dots, E_{1s},\ E_{21}, \dots\dots\dots\dots, E_{2s},\ E31, \dots\dots\dots, E3s \qquad (2.1)$$

Where $E_{ai} = \phi_a V_i \, , a = 1, 2, 3 \; and \; i = 1, 2, \dots \dots, s$

For any vector field Y tangent to M, we have

$$Y = PY + \sum_{i=1}^{3} \sum_{i=1}^{s} W_{bi}(Y) E_{bi} \qquad (2.2)$$

Where $W_{bi}(Y) = g(Y, E_{bi})$ (2.3)

Operating (2.2) by ϕ_a and using (1.2) and (1.5), we get

$$\phi_a Y = \phi_a PY + \sum_{i=1}^{s} \{ W_{bi}(Y) E_{ci} - W_{ci}(Y) E_{bi} - W_{ai}(Y) V_i \} \qquad (2.4)$$

where a, b, c is cyclic permutation of (1,2,3)

Now using (1.10), (1.11) and (2.4) in (1.8), we have,

$$(\tilde{\nabla}_x \phi_a) Y = g(\phi_a X, Y) \xi_a - \eta_a(Y) \phi_a X$$

$$= g(\phi_a X, Y) \xi_a - \eta_a(Y) \{ \phi_a PX + \sum_{i=1}^{s} \{ W_{bi}(X) E_{ci} - W_{ci}(X) E_{bi} - W_{ai}(X) V_i \}$$

On the other hand,

$$(\tilde{\nabla}_X \phi_a) Y = \tilde{\nabla}_X (\phi_a Y) - \phi_a (\tilde{\nabla}_X Y)$$

$$= \tilde{\nabla}_X \left(\phi_a PY + \sum_{i=1}^{s} \{ W_{bi}(Y) E_{ci} - W_{ci}(Y) E_{bi} - W_{ai}(Y) V_i \} \right) - \phi_a \nabla_X Y - \phi_a h(X, Y)$$

$$= \nabla_X \phi_a PY + h(X, \phi_a PY) + \sum_{i=1}^{s} \{ W_{bi}(Y) \big(\nabla_X E_{ci} - h(X, E_{ci}) - W_{ci}(Y) \big(\nabla_X E_{bi} + h(X, E_{bi}) - g \big(\widetilde{\nabla}_X Y, E_{ai} \big) V_i - g \big(Y, \widetilde{\nabla}_X E_{ai} \big) V_i - g(Y, E_{ai}) \widetilde{\nabla}_X V_i \big) - \phi_a P \nabla_X Y \} - \sum_{i=1}^{s} \{ W_{bi}(\nabla_X Y) E_{ci} + W_{ci}(\nabla_X Y) E_{bi} - W_{ai}(\nabla_X Y) E_{bi} - W_{ai}(\nabla_X Y) V_i \} - B_a h(X, Y) - C_a h(X, Y)$$

and separating the normal parts, we get

$$\sum_{i=1}^{s} \eta_a(Y) W_{ai}(X) V_i = h(X, \phi_a PY) - C_a h(X, Y) + \sum_{i=1}^{s} \{ W_{bi}(Y) h(X, E_{ci}) - W_{ai}(Y) h(X, E_{bi}) - W_{ai}(Y) \nabla_X{}^{\perp} V_i - X(W_{ai}(Y) V_i) + W_{ai}(\nabla_X Y) V_i \} \qquad (2.5)$$

for $X, Y \in \lceil(TM)$

using (2.3) in (2.5), we get

$$h(X, \phi_a Y) - C_a h(X, Y) + \sum_{i=1}^{s} W_{ai} (\nabla_X V_i) = 0 \; for \; X, Y \in \lceil(D) \qquad (2.6)$$

Lemma 2.1 If M is a QR-Submanifold of a manifold with a Kenmotsu 3-Structure, than $\xi_a \in \lceil(D), a = 1, 2, 3$

Proof : Let $Z \in D^{\perp}$, then (1.2) and (2.4) give

$$g(\xi_c, Z) = g(\phi_a \xi_b, Z)$$

$$= -g(\xi_b, \phi_a Z)$$

$$= -g(\xi_b, \phi_a PZ + \sum_{i=1}^{s} \{W_{bi}(Z)E_{ci} - W_{ci}(Z)E_{bi} - W_{ai}(Y)V_i\}$$

$$= -\sum_{i=1}^{s} W_{bi}(Z) g(\xi_a, V_i) = 0$$

implying that $\xi_a (a = 1, 2, 3)$ are in D

Lemma 2.2 Let M be a QR-Submanifold of a manifold $\widetilde{M}$ with a Kenmotsu 3-Structure, then we have

$$(\nabla_X B_a)V = -\phi_a\, PA_V X - \sum_{i=1}^{s}\{\, W_{bi}(A_V X)E_{ci} + W_{ai}(A_V X)E_{bi}\,\} + g(\phi_a\, X, Y)\, \xi_a + \eta_a(Y)\phi_a X + A_{C_a V}\, X \tag{2.7}$$

$$(\nabla_X C_a)V = \sum_{i=1}^{s}\{\, W_{ai}(A_V X)V_i - h(X, B_a V) \tag{2.8}$$

for any $X \in \lceil(TM)\ and\ V \in \lceil(TM^{\perp})$

$$h(X, \xi_a) = -\sum_{i=1}^{s} W_{ai}\,(X)V_i\ for\ any\ X \in \lceil TM \tag{2.9}$$

$$h(X, \xi_a) = 0\ for\ any\ X \in \lceil(D) \tag{2.30}$$

where we defined $(\nabla_X B_a)V\ and\ (\nabla_X{}^{\perp} C_a V)$ respectively.

$$(\nabla_X B_a)V = \nabla_X B_a V - B_a \nabla_X V$$

and $\left(\widetilde{\nabla}_x C_a\right) V = \nabla_X{}^{\perp} C_a V - C_a \nabla_X{}^{\perp} V$

Proof: Differentiating (1.13) covariantly along X and using (1.13) and (2.4) and separating the tangent and normal parts we get (2.7) and (2.8) respectively

Putting $Y = \xi_a$ in (1.10) and using (1.9) and (2.4), we get (2.9). Now taking $X \in \lceil(D)$ and using (2.3) we get (2.10)

Theorem 2.1 Let M be a QR-Submanifold of a manifold $\widetilde{M}$ with a Kenmotsu 3-Structure, then

$$h(X, \phi_a Y) = h(Y, \phi_a X) for\ any\ X, Y\ \epsilon \lceil(D), a = 1,2,3 \tag{2.11}$$

$$h(D, D) = \{0\} \tag{2.12}$$

the distribution is involutire (2.13)

Proof: $(2.11) \Longrightarrow (2.12)$

$$h(\phi_3 X, Y) = h(X, \phi_3 Y)$$

$$= h(X, (\phi_1 \otimes \phi_2 - \xi_1 \otimes \eta_1)Y$$

$$= h(X, (\phi_1\ 0\ \phi_2)Y) - h(X, \eta_2(Y)\xi_1)$$

$$= h(\phi_3 X, Y) + \eta_1(X) h(\xi_2, Y) - \eta_2(Y)\, h(X, \xi_1)$$

which with the help of (2.10) give (2.12)

Now (2.12) implies that $h(X, Y) = 0\ \forall\ X, Y \in \lceil(D)\ and$ hence (2.3) and (2.6) gives

$$g(\nabla_X Y, E_{ai}) = 0 \ \forall X, Y \ \in \Gamma(D) \, , a = 1,2,3 \ and \ i = 1, \dots \dots \dots , s$$

Showing that D is involutive

Finally, assuming that D is involutive, we have

$$g([X,Y], E_{ai}) = \ W_{ai}([X,Y]) = 0 \ for \ all \ X, Y \in \ \Gamma(D) \ and \ a = 1,2,3 \ \ and \ i = 1, \dots \dots \dots, s$$

Then (2.6) gives (2.11).

Lemma 2.3 Let M be a QR-Submanifold of manifold $\widetilde{M}$ with a Kenmotsu 3-Structure. Then we have

$$A_{v_i} E_{a_j} \ = \ A_{v_j} E_{a_i} \ for \ a = 1,2,3 \ \ and \ i,j = 1, \dots \dots \dots , s \qquad (2.14)$$

Proof Putting $Y = \ V_i$ in (1.8), we get

$$\widetilde{\nabla}_X(\phi_a V_i) = \ \phi_a \widetilde{\nabla}_X V_i + g(\widetilde{\nabla}_X \phi_a) V_i$$

$$= \ -\phi_a A_{v_i} X - \ \phi_a \nabla_X{}^{\perp} V_i + g(\phi_a X \, , V_i)\xi_a - \ \eta_a(V_i)\phi_a X$$

or

$$\widetilde{\nabla}_X E_{ai} \ = \ -\phi_a A_{v_i} X - \ \phi_a \nabla_X{}^{\perp} V_i + g(\phi_a \ X \, , V_i)\xi_a - \ \eta_a \ (V_i)\phi_a X$$

which on using (1.10) and (1.11) gives

$$h(X \, , E_{ai}) = \ -\phi_a A_{Vi} X + \ \phi_a \nabla_X{}^{\perp} V_i - \nabla_X E_{ai} \qquad (2.15)$$

and hence for any $X \in \ \Gamma(TM)$, we have

$$g\left(A_{V_j} E_{ai} \, , X\right) = g\left(A_{V_i} E_{aj} \, , X\right)$$

from which we get (2.14)

Lemma (2.4) Let M be a QR-Submanifold of a manifold $\widetilde{M}$ with a Kenmotsu 3-Structure, then we have

$$g\left(\nabla_{E_{ai}} E_{a_j} \, , X\right) = g\left(\nabla_{E_{aj}} E_{a_i} \, , X\right) \ \ for \ X \in \ \Gamma(D) \qquad (2.16)$$

$$g\left(\nabla_{E_{ai}} E_{a_j} \, , \xi_b\right) = \sum_{i=1}^{s} W_{bi} \ (E_{a_i}) g \ \left(E_{a_j} \, , V_i\right) = 0 \qquad (2.17)$$

Proof With the help of (1.8), (1.10), (1.11) and Lemma (2.11), we have

$$g\left(\nabla_{E_{ai}} E_{aj} \, , X\right) = g(\nabla_{E_{ai}} \phi_a V_j \, , X)$$

$$= g\left(\left(\widetilde{\nabla}_{E_{ai}} \phi_a\right) V_i \, , X\right) + g\left(\phi_a\left(\widetilde{\nabla}_{E_{ai}} V_i\right), X\right)$$

$$= g(\phi_a E_{ai} \, , V_i)\xi_a - \ \eta_a(V_i)\phi_a E_{ai}, X) + g(\phi_a\left(-A_{v_i} E_{ai} + \ \nabla_X{}^{\perp} V_i\right) , X)$$

$$= \ -g(\widetilde{\nabla}_{E_{ai}} V_j \, , \phi_a X)$$

which with the help of (2.14) gives (2.16)

Putting $X = \ \xi_b$ in (2.16) and using (2.9), we get (2.17). If we define the differential 1-forms $B_{a_{ij}}(X)$ on D by (Bejancu, A. 1986)

$$B_{a_{ij}}(X) = g\left(\nabla_{E_{ai}} E_{a_j}, X\right) \qquad (2.18)$$

for all $X \in \lceil(D), a = 1, 2, 3 \; and \; i, j = 1, \dots \dots \dots \dots, s$

Then (2.16) gives $B_{a_{ij}}(X) - B_{ji}(X) = 0$(2.19)

Theorem 2.2 Let M be a QR-submanifold of a manifold $\widetilde{M}$ with a Kenmotsu 3-Structure. Then the distribution $D^{\perp}$ is integrable if and only if

$$B_{a_{ij}}(X) = 0 \qquad (2.20)$$

Proof we have

$$E_{bi} = \phi_b V_i$$

Operating ϕ_a and using (1.5), we get

$$\phi_a E_{bi} = E_{ci}$$

Similarly, we can show that

$$E_{ai} = \phi_b E_{ci} = -\phi_c E_{bi} \qquad (2.21)$$

By means of (1.8) , (2.16) and (2.21), we obtain

$$g\left(E_{ai} E_{bj}, \phi_c X\right)$$

$$= g\left(\widetilde{\nabla_{E_{ai}}} E_{b_j} - \nabla_{E_{b_j}} E_{ai}, \phi_C X\right)$$

$$= g(\widetilde{\nabla_{E_{ai}}}\left(\phi_c E_{aj}\right) + \widetilde{\nabla_{E_{bj}}}(\phi_c E_{bi}), \phi_c X)$$

$$= g\left(\left(\widetilde{\nabla_{E_{ai}}} \phi_c\right) E_{aj} + \phi_c\left(\widetilde{\nabla_{E_{ai}}} E_{aj}\right), \phi_c X\right) + g\left(\left(\widetilde{\nabla_{E_{bj}}} \phi_c\right) E_{bi} + \phi_c\left(\widetilde{\nabla_{E_{bj}}} E_{bi}\right), \phi_c X\right)$$

$$= B_{a_{ij}}(X) + B_{ji}(X) \; for \; X, Y \in \lceil(D) \qquad (2.22)$$

Now (2.16), gives

$$g\left(\left[E_{ai}, E_{aj}\right], X\right) = 0 \qquad (2.23)$$

Hence (2.22) and (2.23) imply that $D^{\perp}$ is integrable, if and only if $B_{a_{ij}}(X) = 0$

Theorem (2.3) Let M be a QR-Submanifold of a manifold $\widetilde{M}$ with a Kenmotsu 3-Structure. If D is integrable, then the leaf of D are totally geoderic in M if and only if $h(D, D) = \{0\}$

Proof For $, Y \in \lceil(D)$, we have

$$g(\nabla_X Y, E_{ai}) = g(Y, \widetilde{\nabla}_X \phi_a V_i)$$

$$= g\left(Y, \phi_a\left(\widetilde{\nabla}_X V_i\right) + \left(\widetilde{\nabla}_X \phi_a\right) V_i\right.$$

$$= g\left(Y, -\phi_a A_{V_i} X + \nabla_X{}^{\perp} V_i + g(\phi_a X, Y)\xi_a + \eta_a(Y)\phi_a X\right)$$

$$= g(h(X, \phi_a Y), V_i) \qquad (2.24)$$

Putting $y = \xi_b$ in (2.24), we get

$$g\left(\nabla_{\xi_b} Y, E_{ai}\right) = g(h(\xi_b, Y), V_i) = 0$$

Thus we see that the leaf of D are totally geodric in M if and only if $h(X, \phi_a Y) = 0 \; for \; X, Y \in \lceil(D) \; i.e. h(D, D) = 0$

Theorem 2.4 Let M be a QR-Submanifold of a manifold $\widetilde{M}$ with a Kenmotsu 3-Structure. Then the leaf of $D^{\perp}$ are totally geoderic in M if and only if $h(D, D^{\perp})$ = {0}

Proof For any $X \in \lceil(D)$, we have

$$g\left(\nabla_{E_{ai}} E_{aj}\ , X\right) = g\left(E_{ai} \phi_a V_j\ , X\right)$$

$$= g\left(\phi_a\left(\widetilde{\nabla}_{E_{ai}} V_j\right), X\right)$$
$$+ g\left(\left(\widetilde{\nabla}_{E_{ai}} \phi_a\right) V_j\ , X\right)$$

$$= g\left(-\phi_a A_{V_i} E_{ai} + \phi_a \nabla_{E_{ai}}{}^{\perp} V_j\ , X\right)$$
$$+ g(\xi_a, X)\, g\left(\phi_a E_{ai}, V_j\right)$$
$$- g(\phi_a E_{ai}\ , X)\eta_a\left(V_j\ \right)$$

$$= g\left(h(E_{ai}\ , \phi_a\ X)\ , V_j\right) = 0 \qquad (2.25)$$

putting $X = \xi_b$ in (2.25), we get

$$g\left(\nabla_{E_{ai}} E_{aj}\ , \xi_b\right) = g\left(h(E_{ai}\ , \xi_c), V_j\right) = 0$$

(2.26)

Hence (2.25) and (2.26) imply that the leaf of $D^{\perp}$ are totally geoderic in M if and only if

$$h(E_{ai}\ , \phi_a X) = 0\ i.e.\ h(D, D^{\perp}) = \{0\}$$

Proposition: Let M be a QR-Submanifolds of a manifold $\widetilde{M}$ with a Kenmotsu 3-structure. Then the curvature tensor $\widetilde{R}\ of\ \widetilde{M}$ satisfy the following relations:

$$g\left(\widetilde{R}\,(X, E_{ai})X, E_{aj}\right) +$$
$$g\left(\widetilde{R}\,(X, E_{ai})\phi_a X\ , V_j\right) = 0 \qquad (2.27)$$

for any $X\ \epsilon \lceil(D), a = 1,2,3\ and\ i = 1, \ldots\ldots, s$

Proof It is sufficient to prove that (2.23) for a = 1. Using (1.8) and (1.10), we get

$$\left(\widetilde{\nabla}_X \widetilde{\nabla}_{E_{1i}} \phi_1 X, V_i\right)$$
$$= -g\left(\widetilde{\nabla}_X \widetilde{\nabla}_{E_{1i}} \phi_1\ X, \phi_1 E_{1i}\right)$$

$$= -g(\widetilde{\nabla}_X \left(\widetilde{\nabla}_{E_{1i}} \phi_1\right) X$$
$$+\ \phi_1\left(\widetilde{\nabla}_{E_{1i}}\ X\right), \phi_1 E_{1i})$$

$$= -g\left(\left(\widetilde{\nabla}_X \phi_1\right)\left(\widetilde{\nabla}_{E_{1i}}\ X\right), \phi_1 E_{1i}\right)$$
$$- g\left(\phi_1 \widetilde{\nabla}_X \widetilde{\nabla}_{E_{1i}}\ X, \phi_1 E_{1i}\right)$$

$$= -g\left(\phi_1 \widetilde{\nabla}_X \widetilde{\nabla}_{E_{1i}} X, E_{1i}\right)$$

from which we get

$$g\left(\widetilde{\nabla}_X E_{1i} \phi_1 X\ , V_i\right) = -g\left(\widetilde{\nabla}_X \widetilde{\nabla}_{E_{1i}} X\ , E_{1i}\right)$$

(2.28)

Similarly, we have

$$g\left(\widetilde{\nabla}_{E_{1i}} \widetilde{\nabla}_X \phi_1\ X\ , V_i\right) = -g\left(\widetilde{\nabla}_{E_{1i}} \widetilde{\nabla}_X X\ , E_{1i}\right)$$

(2.29)

$$g\left(\widetilde{\nabla}_{[X, E_{1i}]} \phi_1 X\ , V_i\right) = -g\left(\widetilde{\nabla}_{E_{1i}} \widetilde{\nabla}_X X\ , E_{1i}\right)$$

(2.30)

Thus (2.27) is obtained by using (2.28) , (2.29) and (2.30) in the formula for Curative

Note Bejancu, A (1986) proved this proposition for a QR-Submanifold of a quaternion Kaehlerian manifold by taking the submanifold to be totally umbilical but in our case this condition is not necessary.

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