

The Graph Theory: Teaching of Graphs in the Contexts of Number Concept and Blended Learning in the Intermediate Phase Classroom in Johannesburg North District in Gauteng Province in South Africa

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Abstract:

The graph theory has become so popular in learning number concepts such as integers. Researchers have explored the use of the graph theory nationally and internationally. The Graph theory is a mathematics strands. The concept of graph is difficult for some learners but for some it could be so fascinating. The purpose of this paper was to explore how best the graph theory could be used to teach relationships of algebra and other suitable contexts to teach problem solving. The other objective was to explore the connections between the graph theory and other sciences. The study is part of the thesis carried out in 2019, "the preferred contexts in learning number concept integers in the intermediate phase in Johannesburg North". The findings show that the graph concept theory can help the learners understand several concepts using the graph theory. The graph theory is a powerful tool that can be used

to learn problem solving, communication, reasoning, connection and representation.

Key Words

Bar graph, Pictograph, histogram, linear graph, double bar graph, algebra, blended learning, frequency table. Table of information.

Introduction

The origins of graph theory can be traced back to Euler's work on the Konigsberg bridges problem (1735), which subsequently led to the concept of an eulerian graph. The study of cycles on polyhedra by the Revd. Thomas Penyngton Kirkman (1806–95) and Sir William Rowan Hamilton (1805–65) led to the concept of a Hamiltonian graph (Gribkovskaia, Halskau, & Laporte, 2007). Researchers have books on graph theory, for example, *Graph Theory* by Frank Harary, (1969). It is believed the word graph originated from French *-graphie*, from Late Latin -

graphus]which means the instrument for making or transmitting records or images. According to Merriam a graph a diagram (such as a series of one or more points, lines, line segments, curves, or areas) that represents the variation of a variable in comparison with that of one or more other variables. The second definition of a graph is the collection of all points whose coordinates satisfy a given relation (such as a function). Data can be presented and Bar graphs, line graphs, and pie charts are useful for displaying categorical data. Graph theory is a field of mathematics that looks to study objects called graphs. The ideas and understanding gained from studying graphs can be applied to many other problems (Faculty of Mathematics, 2020) Thus graphs are integral in the learning of mathematical skills.

The National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics (NCTM) provides *Principles and Standards for School Mathematics (PSSM)* which show the guidelines set in 2000 by the council which set the recommendations for mathematics educators (Wikipedia, the free encyclopedia, 2000) These strands are divided into mathematics content such as Number Operations, Algebra, Geometry, Measurement, Data Analysis & Probability and processes. These processes (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) are specific expectations for learners learning are described for ranges of grades (preschool to 2, 3 to 5, 6 to 8, and 9 to 12). The South African policy document guides what should be taught as Numbers, Operations and Relationships Patterns, Functions and Algebra, Space and Shape (Geometry), Measurement and Data handling (Mathematics English book: Rainbow workbook, 2018) It is from this background that this

study focused on one aspect data handling focusing on the graph theory.

The United Nations Educational, Cultural and Scientific Organisation (UNESCO)

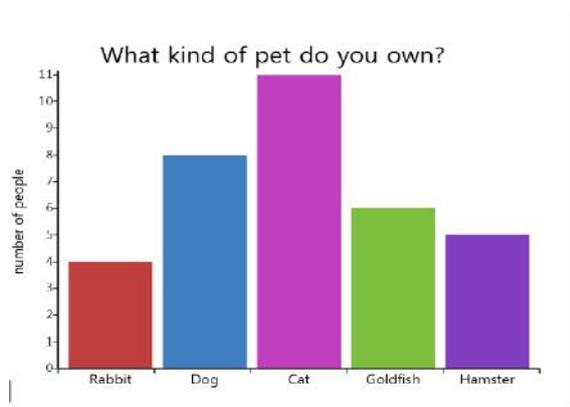
The (UNESCO) encourages Teaching Tolerance (2019), students consider different ways that young people have used the Internet to work toward positive social change and the strengths and weaknesses of online activism. (UNESCO, 2019) This study focuses on blended learning through the use of graphs. Education is UNESCO's top priority because it is a basic human right and the foundation on which to build peace and drive sustainable development. UNESCO is the United Nations' specialized agency for education and the Education Sector provides global and regional leadership in education, strengthens national education systems and responds to contemporary global challenges through education with a special focus on gender equality and Africa. South Africa as a member state confides to the education requirements of the mother body that is the UNESCO. Both (NCTM) and (UNESCO) agree on the use of graphs in teaching mathematics.

The types of different graphs

There are different kinds of graphs namely a line graph, a bar graph, a pie chart, a pictograph, a histogram and these are used to show different kinds of data. Bar graphs, line graphs, and histograms have an x- and y-axis. The x-axis is the horizontal part of the graph and the y-axis is the vertical part. The Department of Basic Education (2018) mentions the bar graph, the double bar graph, the histogram, the frequency graph and the pie chart. These graphs are explained individually in the paragraph below beginning with the bar graph.

The Bar graph

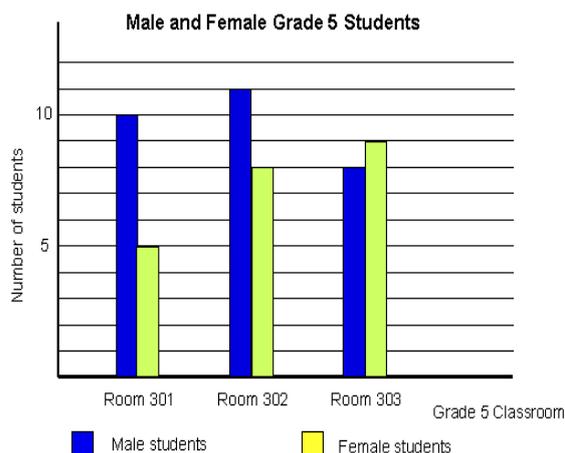
The bar graph is a visual display that compares the frequency occurrences of different characters of data (Basic Education Grade six, 2018, p. 148). A bar graph is composed of discrete bars that represent different categories of data. The length or height of the bar is equal to the quantity within that category of data. Bar graphs are best used to compare values across categories. Bar graphs usually show categorical data - e.g. energy use of a fridge, television, geyser. Bar graphs are used to compare variables. Below is an example of a bar graph that can be used for different grades.



The next graph explained is the double bar graph

The double bar graph

Research shows that a double bar graph is similar to the regular bar graph, but gives two pieces of related information for each item in the vertical axis instead of one. The double graph well compares the related groups of data (Basic Education Grade six, 2018, p. 152). A double graph can be used for example when recording children attending a function and parents of given days and the use of colour is used to indicate the children, for example, red and blue for the adults. Below is an example of a double bar graph that can be used in the intermediate phase.

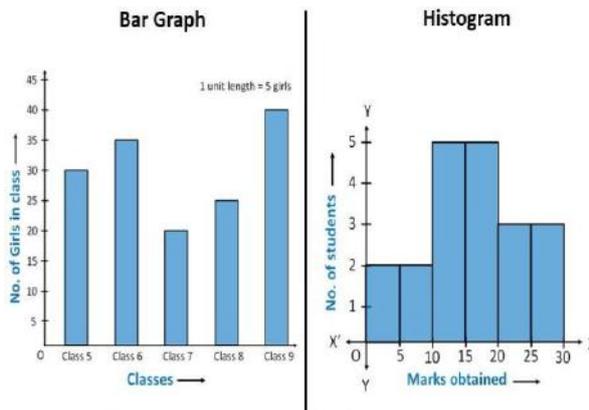


The next mentioned graph is the histogram.

The Histogram

The histogram is a particular kind of bar graph that summarizes data points falling in various ranges. The main difference between a histogram and a bar graph is that the bar graph shows the frequency of each item/element of data, while the histogram shows the range. The histogram must touch because the data elements are reading numbers that are grouped to form the continuous range from the left to the right (Basic Education Grade six, 2018, p. 152). The histogram should be connected. Histograms usually show continuous data that represents measured quantities. Histograms are used to show the distribution of variables. The bars in bar graphs are usually separated. In histograms, the bars are adjacent (next) to each other. Sometimes bar graphs may have no space between the bars. Histograms are never drawn with spaces between the bars - the bars are next to each other.

The difference between a bar graph and a histogram



Research shows that there is a difference between a histogram and a bar graph. It is noted that in the graph the bars are equally spaced and on the Y-axis, we can have numbers and on the x-axis, we have data that can be anything. In the histogram, the bars are fixed and in the Y-axis we have to have numbers that are continuous and will always be a number.

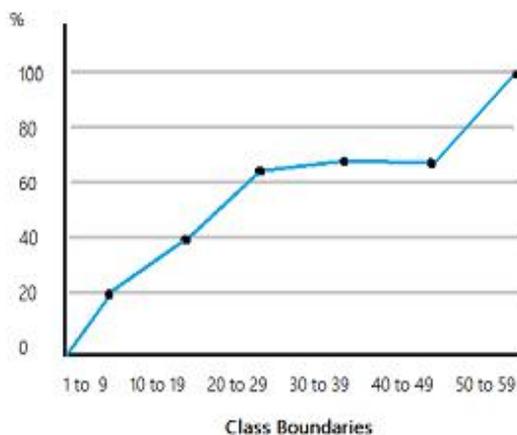
The next graph is the frequency graph.

The frequency graph.

The next graph is the frequency graph represents data with a well-defined peak that is close to the median and the mean, while the “outliers” are of relatively low frequency (Basic Education Grade six, 2018, p. 160)The data table lists a set of scores and they are the frequency.

The frequency Graph

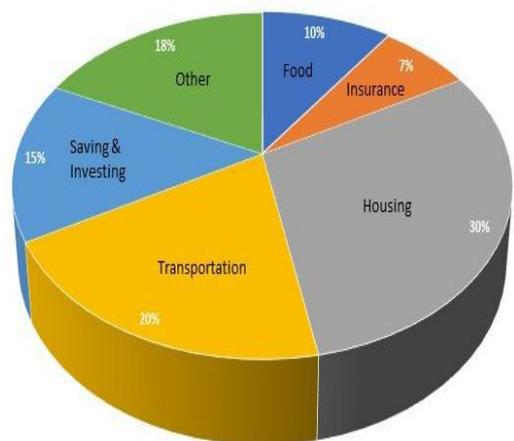
The Population in Marufu District that Voted in an election in 2021.



The next graph is the pie chart

The Pie Chart

A pie chart is a circular chart used to compare parts of the whole. It is divided into sectors that are equal in size to the quantity represented. Each sector visually represents an item or data set. The size of the sector is in proportion to the number of items as a percentage or fraction of the total data set. The researchers believe that pie charts are useful to compare budgets and financial information. (Basic Education Grade six, 2018, p. 164)



The next graph is the pictograph.

Pictographs

A pictograph is a way of showing information using pictures or images. This is a simple way of introducing learners to presenting information in tables and graphs later on (D.B.E, 2018)The pictographs are usually used in the early grades as it is easy for them to count and colour the pictures they learn data handling.

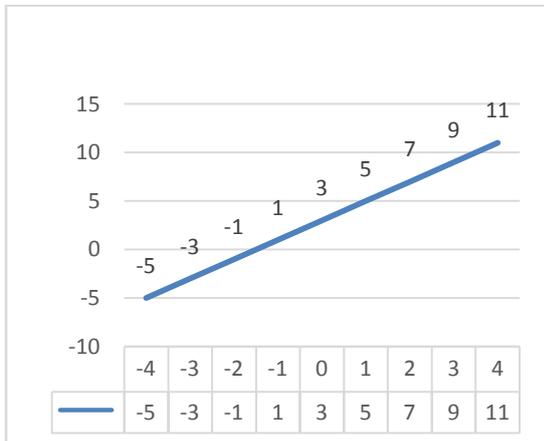
The following is a pictograph showing learners and the mode of transport used for coming to school.

The learners and transport used to come to school

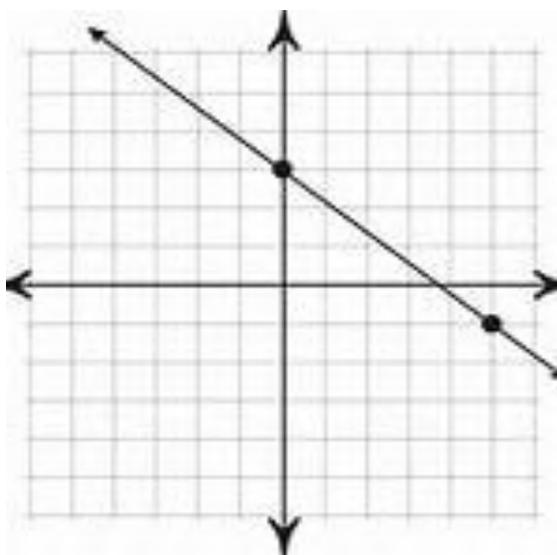
| | |
|---------|---|
| Bus |  |
| Car |  |
| Cycle |  |
| On foot |  |

The next graph is the linear graph.

The graph for $y = x + 3$



Or the following



The vertical being the y axis and the horizontal line is the x-axis. The next graph is the curved line graph.

The Curved Line Graph

If a position graph is **curved**, the slope will be changing, which also means the velocity is changing. Changing velocity implies acceleration. So, curvature in a **graph** means the object is accelerating, changing velocity/slope.

Literature Review in a South African Perspective.

Graphs theory

The graphs are learnt in grade one under the topic collecting and organizing, sort the following given objects for example leaves by making a drawing. The curriculum, in all grades (Department of Basic Education, 2021, p. 92) grade 1, (Department of Basic Education, 2021, p. 119) grade 4 (Department of Basic Education, 2021, p. 146) grade 5 (Department of Basic Education, 2021, p. 268) grade 6 support the use of graphs through the Curriculum Assessment Policy Statement (CAPS).

Grade one

In grade one, the children are given objects of yellow-orange and green. These could be boxes, circles or different shapes. The learners sort the objects by colour (Department of Basic Education, 2021, p. 92).

The learners are expected to draw a pictograph.

The learners answer the questions

How many yellow objects are there?

How many oranges objects are there?

How many green objects are there?

Adapted from Department of Basic Education Mathematics in English books 1 and 2 (2021).

Grade two

The graph theory is used in grade two in a South African classroom. In grade two (2) the learners collected coins. These learners were Siphon, Jane, Peter, Thabo, Mavis and Sphu. The learners draw a pictograph for example Sphu collected.

| Sorting My coins | | | | | |
|------------------|-----|-----|-----|----|-------|
| 6 | | | | | |
| 5 | | | | | |
| 4 | | | | | |
| 3 | | | | | |
| 2 | | | | | |
| 1 | | | | | |
| | 50c | 20c | 10c | 5c | R1,00 |

This is adapted from Brandt, Dodd, Pereira and Sorour (2011)

The theory of graphs in grade 2 uses data collection techniques using pictographs and dice playing the recording in the columns (Brandt, Dodd, Pereira, & Sorour, 2011)The theory is supported by the Department of Basic Education which contains that the data should be collected as fruits, sort the fruits and write the totals in a box. The learners' pictographs showing the sorted fruits. They find which fruit do we have most. Which fruit do we have least? shapes. The learners sort the objects by colour (Department of Basic Education, 2021, p. 64)

The fruits sorted by the learners

| | | | | | | |
|------------------|----|--|---------|------|-------|--------|
| Number of fruits | 10 | | | | | |
| | 9 | | | | | |
| | 8 | | | | | |
| | 7 | | | | | |
| | 6 | | | | | |
| | 5 | | | | | |
| | 4 | | | | | |
| | 3 | | | | | |
| | 2 | | | | | |
| | 1 | | | | | |
| | | | berries | Pear | Apple | Banana |
| Types of fruit | | | | | | |

Adapted from Department of Basic Education grade 2(2021).

Grade 6

The graph theory is used in grade six (6), according to

the Department of Education the learners draw up the tally marks and frequency tables. They record information such as the healthy breakfast. The learners make ticks of the food eaten during breakfast. The learners do the tallies and then the graphs and comparisons (Department of Basic Education, 2021, p. 76).

The learners' group and order data on temperature using ranges and draw histograms after completing the table of frequencies. They draw and analyze pictographs, pie charts and column graphs. The learners in grade 6 calculate the mean, mode and median of given data. It seems the graph theory broadens so much in grade six.The learners are expected to be able to read and interpret the bar graphs and pie charts.

Grade Seven (7)

The graph theory is implemented in grade seven (7) by the Department of Basic Education by looking at the data handling cycle and describe it. The learners should start with the question, collect the data, organize and record data, represent data into graphs, interpret the graph answer questions, predict and ask new questions.

The learners should be able to interpret graphs on time and temperature.

According to (DBE) the grade seven learners should be able to come up with a hypothesis and before collecting the data the grade seven learners must be able to ask correct questions.

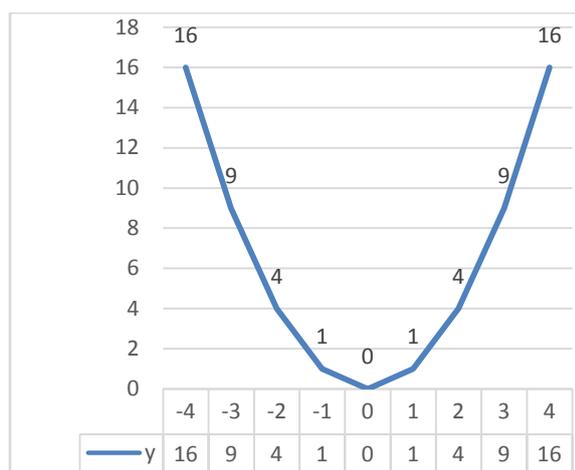
The example of the hypothesis is every grade seven learner should be able to work with square roots.

$$y = x^2 + 2$$

The square root graph

The Graph

$$y = x^2$$



Designed by the researcher Marufu (2021)

Problem Statement

South Africa suffers from weak educational quality, despite relatively high levels of public spending on education (Mlachila & Moeletsi, 2019). Many learners were left out or excluded in systemic education for one reason or the other although South African education entered the new era from 1994 when democracy came into being. The new Constitution of the Republic of South Africa (Act No 108 of 1996) was adopted in 1996 and its supreme law of the country and binds all schools to nurture and protect fundamental human rights enshrined in the bill of rights.

Research Method

The research was carried out on 29 primary school learners. The sample size was $N=29$. The data were collected from 29 primary school learners using the qualitative method research approach. The sample was conveniently sampled because the researcher teaches a primary school in Johannesburg North District where the research was done, it was easier to obtain the sample from this aforementioned school. The interviews were done after school in room 23 where the researcher was based. The interviews followed a set interview by the researcher, the parents

and the learners. The parents had to make transport arrangements for dropping and picking the learners. Three learners from each grade were interviewed and then the results were analysed using the theory graph content analysis. In this study, the theory of graph content analysis is believed to be a powerful tool in teaching number concepts such as integer recognition and integer operations. The learners were interviewed in room 23 where the researcher was stationed. The learners used blended learning to consolidate the graphs by making graphs using the computers and writing questions and answers using the computer and printing the graphs for record purposes.

Ethical Clearance

The ethical clearance measures were taken following the 2019 guidelines of the Tshwane University of Technology and all the names of the learners in the article are pseudonyms so that the learners were protected against the rights of children. The parents of the learners signed the consent forms and the learners signed the assent letters. No benefits were promised to both the parents and the learners for accepting to provide information in the study.

Analysis

The interviews were held in room 23 in the class where the researcher is stationed. The learners were given questions specifically for the grade because the policy curriculum shows different theories of the graph at different levels with the guidelines of the Basic Education in teaching across the board. The analysis was sorted as per grade following the sequences of teaching the concepts of the graph.

Responses from three Grade one learners

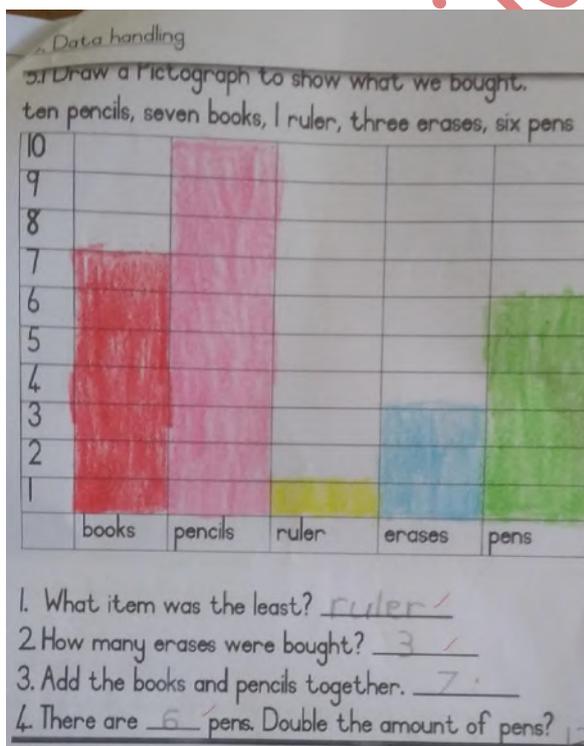
Responses from Emy

In the interview, the learners were given a chart with pictures of objects. They were asked to colour the

objects and count them. The learners were asked to place the number of the object using colour on a vertical axis of a graph. The first grade one learner to be interviewed was Emy, Emy from grade one was 6 years old. She coloured the objects correctly, coloured the bars correctly in line with the correct picture. Below is the presentation was given by Emy. responses were presented in pictograph form.

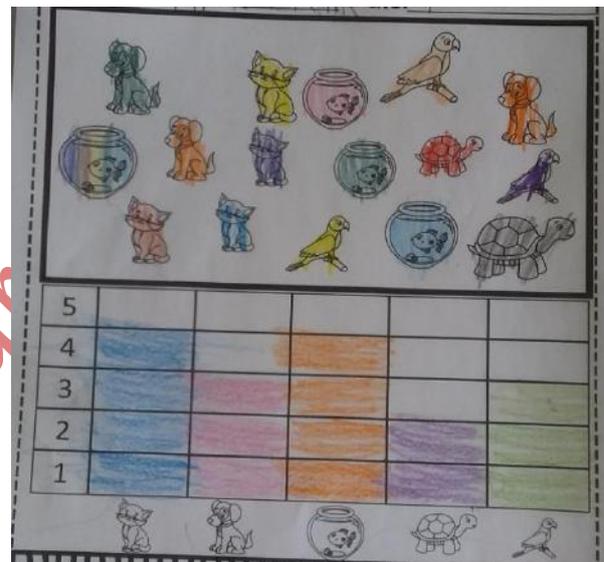
Responses from Adria

The grade one learners were interviewed in room 23 and the second grade one learner to be interviewed was Adria. Adria, a girl was 7 years old. Adria was given different objects with different quantities to sort and count. Considering the responses given by Adria, she was able to count the objects, record them and write the numbers correctly. Adri was able to sort the items, colour the bars in a graph in the graph and analyse the information in the graph using language. The graph theory in this case was able to hence, sorting, counting, colouring and analysing in using the pictograph. The presentations below were given by Adria in the interview:



Responses from Chad

The grade one learners were interviewed in room 23 and the second grade one learner to be interviewed was Chad. Chad, a girl was 7 years old. Chad was given different objects with different quantities to sort and count. Considering the responses given by Chad, she was able to count the objects, record them and write the numbers correctly. Adri was able to sort the items, colour the bars in a graph in the graph and analyse the information in the graph using language. The graph theory in this case was able to hence, sorting, counting, colouring and analysing in using the pictograph. Below is the pictograph presented by Chad.



Considering the responses given by Chad, the graph theory could be used as a powerful tool to interpret given information into data. According to Chad all questions and interpretations were done correctly using the pictograph.

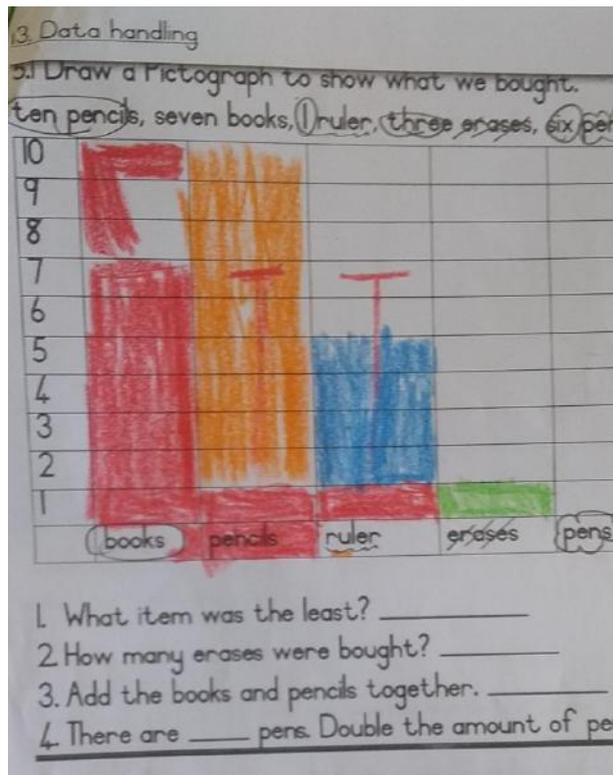
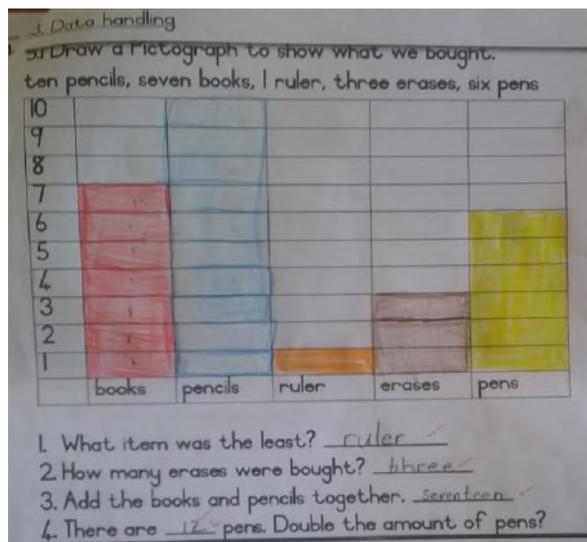
Grade two responses

The grade two learners were interviewed in room 23.

Responses from Gef

The grade two learners interviewed were Gef, Caity and Arry. The learners were given a statement, draw a graph to show the things we bought and the things

were ten pencils, seven books, one ruler three erases and six pens. Considering the responses given by Gef, a boy aged 7 from grade two, we bought 10 books, ten pencils, 5 rulers, one erase and no pens. Gef was unable to interpret the information given. Below is the graph provided by Gef. The given questions that were meant to analyse the data using the graph were never attended to. The presentation below was given by Gef:



Responses from Caity

The learners were given a statement, draw a graph to show the things we bought and the things were ten pencils, seven books, one ruler three erases and six pens. Considering the responses given by Caity when given the instruction, we bought seven books, ten pencils, one ruler, three erases six pens. All the bars were coloured correctly and all the questions were answered correctly following the quantities given on the vertical axis. Below are responses given by Caity. Caity successfully used the graph theory to analyse the given data. Below is the response was given by Caity.

Responses from Arry

The learners were given a statement, draw a graph to show the things we bought and the things were ten pencils, seven books, one ruler three erases and six pens. Considering the responses given by Arry when given the instruction, we bought seven books, ten pencils, one ruler, three erases six pens. All the bars were coloured correctly and all the questions were answered correctly following the quantities given on the vertical axis. Below are the responses given by Arry. The graph theory was used as a powerful tool to analyse the given information in grade two.

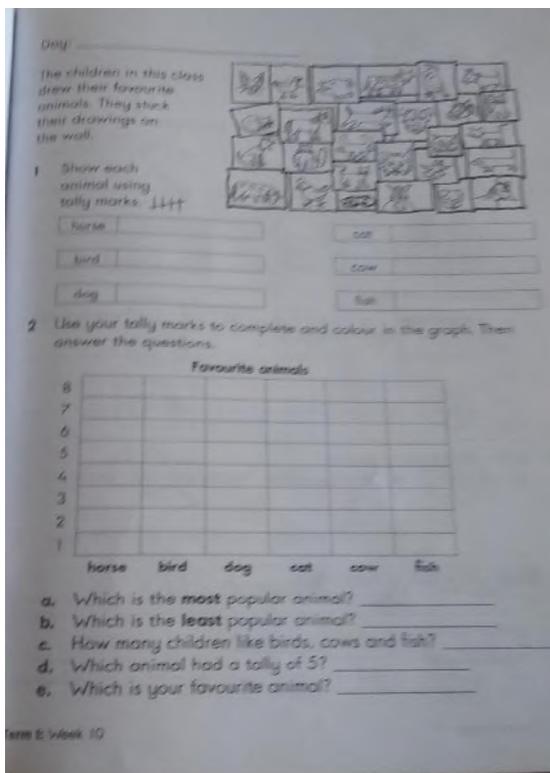
The responses from the grade two learners clearly show that the learners could use the graph theory to interpret given information. The next section analyzes the graphs presented by grades three.

The Responses are presented by grade three.

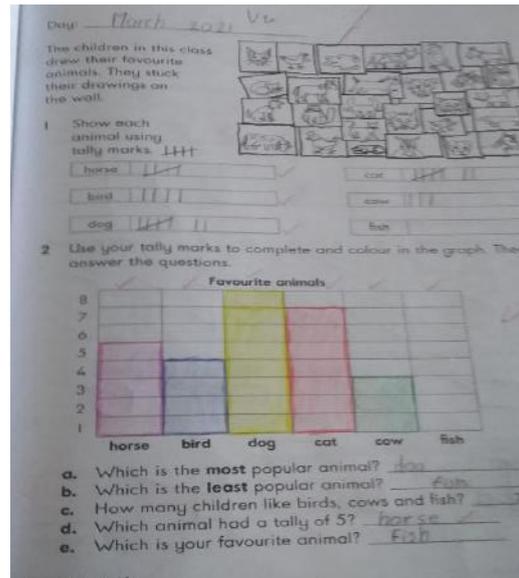
The grade three learners were interviewed in room 23 and the grade learners interviewed were in Grade three. The grade three learners were given 3 different animal pictures to identify, count pictures, make tally marks, draw graphs and answer the questions related to the graphs. The learners were asked to analyze a chart on animal pictures. They were then asked to draw a graph and answer the questions that followed

the graph. The learners were given pictures of horses, birds, dogs, cats, cows and fish. The three learners who were asked to analyze the picture chart were Ci, Vu and Eli. Ci is a girl from Grade 3.

The first learner to be interviewed was Ci, a girl from Grade 3 aged 9 and she could not answer even one question. She just smiled no matter how the interviewer tried to simplify the questions. She came out with a blank sheet with no answers at all. Below is the blank page handed in by Ci. She was interviewed in room 23 where the researcher was stationed.

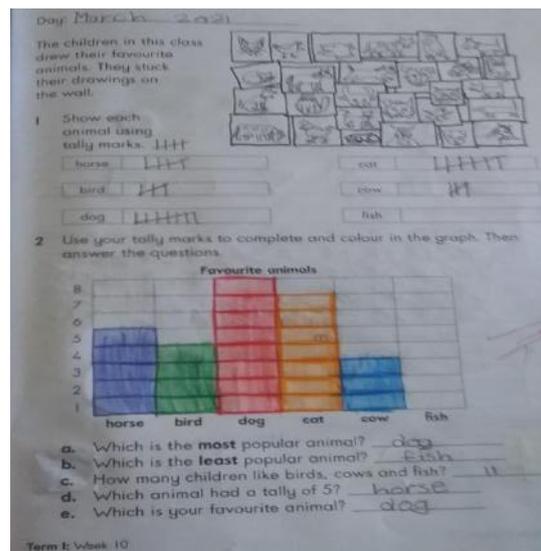


The second learner to be interviewed was Vu, from Grade 3, a boy aged 9. According to the responses given by Vu, all the tallies were correct, horses 5, cats 7, birds 4, cows 4, dogs 7 except the last one which was not answered correctly. The fish was indicated as zero instead of 4. Below are the responses provided by Vu.



The third grade 3 learner to be interviewed was Eli, a boy from grade 3 aged 9 years. The tallies give for horses was 5 and the cats 7 but the tally grouping was not correctly done, the birds were tied up when the tallies were only four and also three tallies were tied up. The counting skills were not correctly mastered in the case of Eli, whose most tallies were incorrect. The bar graph structure was done with the wrong tallies. According to the responses given by Eli, there were all animals recorded but sometimes in the wrong way.

Below was the response provided by Eli



The Responses were presented by grade four.

The following section shows the graph interpretation shown by the grade four (4) learners in the interviews in room 23. The grade four learners were given

different mixed fruit pictures. They were asked to sort the fruits and count them into sets or circles. These were naartijies, apples, bananas and strawberries. Four learners were interviewed and these were Kenzo, Ethas, Sibiu and Omi (all these are pseudonyms for the protection of personal identity). The following are the responses according to Kenzo.

When asked to sort out the fruits Kenzo, a boy aged 9, from grade 4 gave the following information. He sorted the fruit circles as follows, there were 12 naartijies, 7 apples, 9 bananas and 15 strawberries. When asked the question, How many naartijies are there? He wrote (10). How many apples are there? He wrote (7). How many bananas are there? He wrote (15). Considering the sorting done by Kenzo, he could place the fruit in the correct sets and has correctly counted the fruit from the initial set. Kenzo could analyse the data using one number answer and it could be picked that he does not use sentences in answering questions.

Kenzo analysis the fruit using the graph theory, he draws 12 naartijies ,9 bananas in the correct positions, 15 strawberries and 7 apples. According to the data provided by Kenzo, the graph theory was correctly used to analyse the data although the graph theory had some guided from the researcher. The following are the responses from Ethas.

Responses from Ethas

The following are the responses according to Ethas. When asked to sort out the fruits Ethas, a boy aged 9, from grade 4 gave the following information. He sorted the fruit circles as follows, there were 12 naartijies, 7 apples, 9 bananas and 15 strawberries. When asked the question, How many naartijies are there? He wrote (10). How many apples are there? He wrote (7). How many bananas are there? He wrote (15). Considering the sorting done by Ethas, he could place the fruit in the correct sets and has correctly

counted the fruit from the initial set. Ethas could analyse the data using one number answer and it could be picked that he does not use sentences in answering questions. The shape of all the fruit was round including the bananas.

Ethas analysis the fruit using the graph theory, he draws 12 naartijies ,9 bananas in the correct positions, 15 strawberries and 7 apples. According to the data provided by Ethas, the graph theory was correctly used to analyse the data although the graph theory had some guidance from the researcher. Below are the responses provided by Sibiu.

Responses from Sibiu

The following are the responses according to Sibiu. When asked to sort out the fruits Sibiu, a girl aged 9, from grade 4 gave the following information. He sorted the fruit circles as follows, there were 12 naartijies, 7 apples, 9 bananas and 15 strawberries. When asked the question, How many naartijies are there? She wrote (10). How many apples are there? She wrote (7). How many bananas are there? She wrote (15). Considering the sorting done by Sibiu, he could place the fruit in the correct sets and has correctly counted the fruit from the initial set. Sibiu could analyse the data using one number answer and it could be picked that he does not use sentences in answering questions. The shape of all the fruit was round including the bananas.

Sibiu analysis the fruit using the graph theory, he draws 12 naartijies ,9 bananas in the correct positions, 15 strawberries and 7 apples. According to the data provided by Sibiu, the graph theory was correctly used to analyse the data although the graph theory had some guidance from the researcher. Below are the responses provided by Sibiu.

Grade 5 responses to the graph theory

The next section analysis the work from the grade 5 (five) learners on the topic of graph theory as a powerful tool for learning mathematical concepts. The

learners interviewed were Mphos, Sekhas and Tallies.

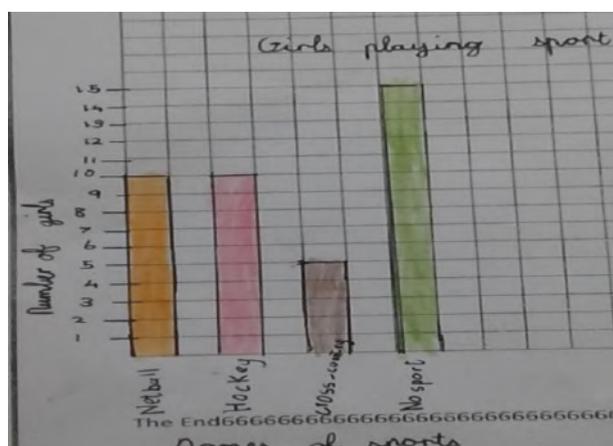
In the interview the learners were given information, they drew tallies, drew the graph, asked questions and answered the questions from the graph. The objective was to explore what skills are obtained in the grade 5 class by using the graph theory.

The responses from Mphos

The first grade five learner to be interviewed was Mphos, Mphos, a girl from grade 5 who was 10 years old. Considering the responses provided by Mphos the graph theory was explained clearly. The grade 5 learners were given different types of games in which the girls were to choose games of their own choice. The given games were netball, hockey and cross-country. The responses given by Mphos show 10 girls voted for netball, 10 girls prefer cross-country and 15 girls do not like any sport at all. The graph has a topic, girls playing sport. The number of girls who like each sport is shown on the vertical Y-axis and the number of girls who like the sport is shown using tallies. On the x-axis, the types of the preferred sports are shown and well labelled. Below is the presentation was done by Mphos

The responses from Sekhas

The first grade five learner to be interviewed was Sekhas, Sekhas a girl from grade 5 was 10 years old. Considering the responses provided by Sekhas the graph theory was explained clearly. The grade 5 learners were given different types of games in which the girls were



to choose games of their own choice. The given games were netball, hockey and cross-country. The responses given by Sekhas show 10 girls voted for netball, 10 girls prefer cross-country and 15 girls do not like any sport at all. The graph has a topic, girls playing sport. The number of girls who like each sport is shown on the vertical Y-axis and the number of girls who like the sport is shown using tallies. On the x-axis, the types of the preferred sports are shown and well labelled.

The responses from Tallies.

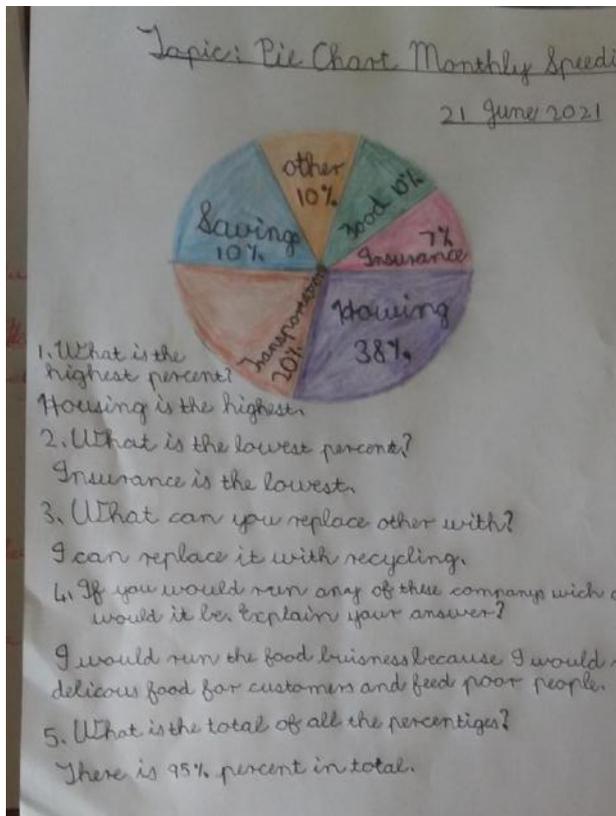
The third grade five learner to be interviewed was Tallies, Tallies a girl from grade 5 was 10 years old. Considering the responses provided by Sekhas the graph theory was explained clearly. The grade 5 learners were given different types of games in which the girls were to choose games of their own choice. The given games were netball, hockey and cross-country. The responses given by Tallies show 10 girls voted for netball, 10 girls prefer cross-country and 15 girls do not like any sport at all. The graph has a topic, girls playing sport. The number of girls who like each sport is shown on the vertical Y-axis and the number of girls who like the sport is shown using tallies. On the x-axis, the types of the preferred sports are shown and well labelled.

The budget pie-graph

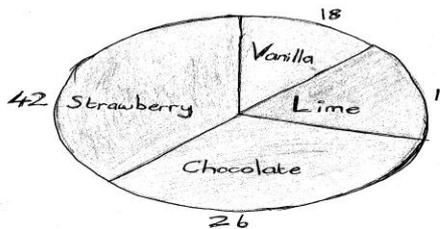
The grade five learners were given a pie chart and they were asked to draw the pie chart and write their questions and answers from the pie chart. The first learner to be interviewed was Caydee, Caydee, aboy aged 11 from grade 5 was interviewed in room 23. The following were the responses from Caydee. The pie-chart was given a topic and all the aspects were coloured using a different colour. Considering the responses given by Caydee, questions were written but were not very correct, for example, the first question was, what was the highest percert, and the

answer was correct as housing has the highest percentage. The second question is what is the lower percent and insurance is the lowest percentage. The third question is if you are asked to do business which business would you do? According to Caydee, he does the food business and the reason is he would prefer to prepare delicious food. The totals of the graph do not add up to one hundred percent. Below are the responses given by Caydee.

The responses of Caydee

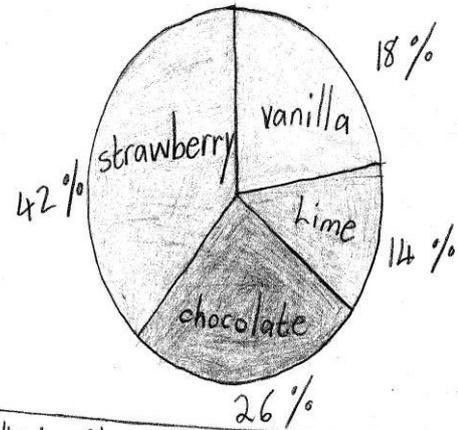


The following were the responses given by



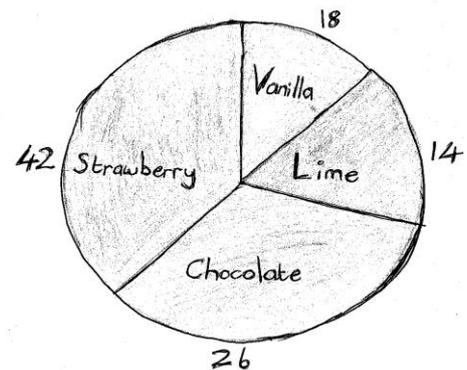
- a) The favorite ice-cream in grade 6 is Chocolate Strawberry
- b) The least favorite ice-cream in grade 6 is Lime.
- c) The difference between Strawberry and Lime $42 - 14 = 28$
- d) $\frac{42}{100}$, $\frac{18}{100}$, $\frac{14}{100}$, $\frac{26}{100}$

The following are the responses from Dua



- 1. Which flavour of ice-cream is the most popular?
Answer: Strawberry ice-cream is the most popular.
- 2. Which flavour of ice-cream is the least popular?
Answer: Lime is the least favourite ice-cream.
- 3. What is the difference between the chocolate and vanilla ice-cream?
Answer: The number 8 is the difference between the chocolate and vanilla.

The following responses were given by Yottee



- a) The favorite ice-cream in grade 6 is Chocolate Strawberry
- b) The least favorite ice-cream in grade 6 is Lime.
- c) The difference between Strawberry and Lime $42 - 14 = 28$
- d) $\frac{42}{100}$, $\frac{18}{100}$, $\frac{14}{100}$, $\frac{26}{100}$

The Responses were presented by the grade six learners (Equation $y = x + 1$).

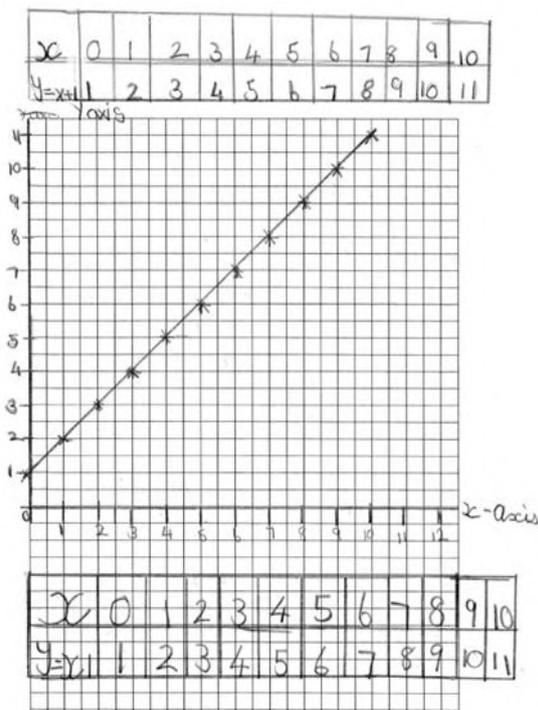
The responses from Lee.

The interviews were held in room 23 in the class where the researcher is stationed. The grade six learners were given the equation $y = x + 1$. Four learners Lee, Cassy, Anto and Sine (pseudonyms) were interviewed in this graph theory. The first learner to be interviewed was Lee. Lee, a boy from grade six was aged 12 years gave the response to the equation $y = x + 1$. He copied and completed the table of information as follows.

| | | | | | | | | | | | |
|-------------|---|---|---|---|---|---|---|---|---|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $y = x + 1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

According to Lee, the graph has the y-axis and the x-axis and all the values are inserted equidistant from each other. The plots of the graph are placed in correct positions. Below is the graph produced by Lee following the given equation. Although Lee could draw this graph, he could name the type of graph he drew. He could not ask questions relating to the graph. Below is the graph presented by Lee.

The graph presented by Lee



Considering the graph produced by Lee the graph had no title but the table of information and the plots were consisted with the graph theories. Lee used the concepts from the graph theory accurately and was not able to ask questions from the graph.

The responses from Cassy.

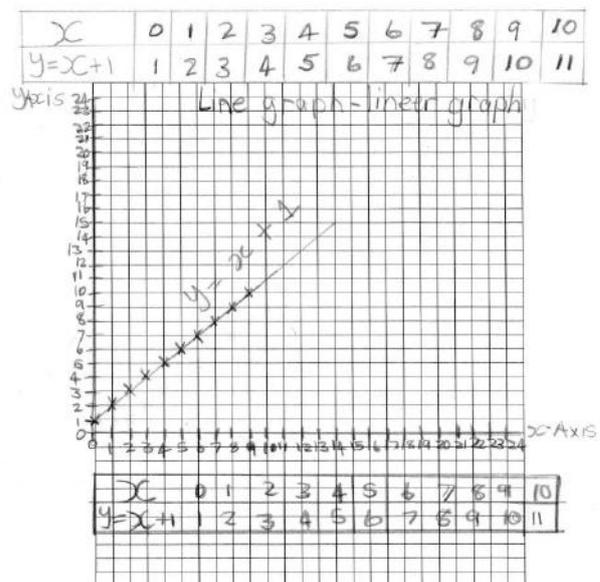
The second graph from the grade sixes was from Cassy. Cassy a girl from grade 6 is aged 12 years. When given the equation $y = x + 1$, she drew up a

table of information and fill in all the missing values as follows:

| | | | | | | | | | | | |
|-------------|---|---|---|---|---|---|---|---|---|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $y = x + 1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

She plotted 10 points and the eleventh plot was not included. She drew the y axis and the x-axis and named the graph line of linear graph. The graph below shows the work presented by Cassy. According to the graph presented by Cassy, the graph presented by Cassy clearly shows that a mathematical concept, the equation could be learnt from the graph perspective in the intermediate phase.

The Responses of Cassy



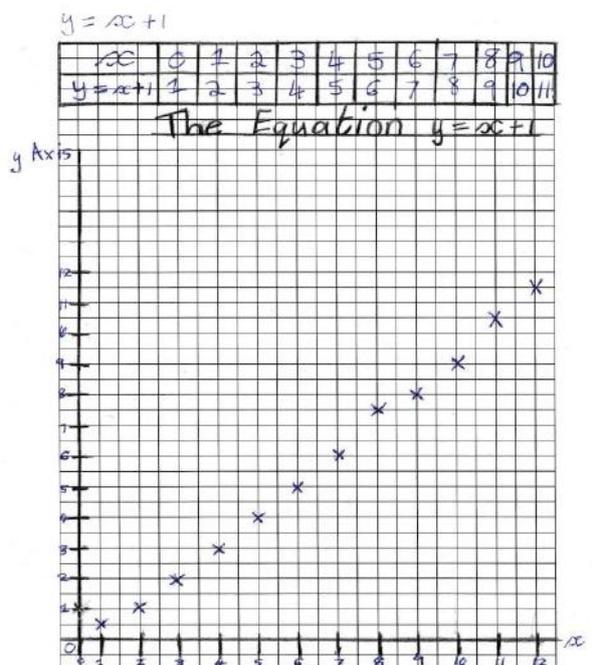
Considering the graph produced by Cassy the title is line graph or linear graph and the equation is given. The data in the information is completed correctly. The graph theory/concept was used as a tool for interpreting the concept of equations. For Cassy, the plots were consistently used the graph theory accurately although there were no questions and answers provided from the graph.

The responses from Anto.

The interviews were held in room 23 in the class where the researcher is stationed. The grade six learners were given the equation $y = x + 1$. Four learners Lee, Cassy, Anto and Sine (pseudonyms) were interviewed in this graph theory. The third learner to be interviewed was Anto. Anto, a girl from grade six was aged 12 years gave the response to the equation $y = x + 1$. She copied and completed the table of information as follows.

| | | | | | | | | | | | |
|-----------|---|---|---|---|---|---|---|---|---|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| y = x + 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

The responses from Anto



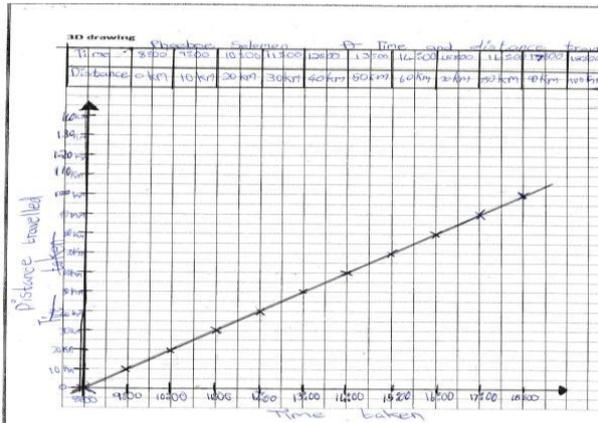
Considering the graph presented by Anto the Y-axis and the x-axis were correctly calculated. The axis was correctly positioned. The graph produced shows that the plotting of the positions of co-ordinates was not accurate and the line of the equation was not drawn. The graph theory presented by Anto shows that some aspects of the theory were not consistent and were not adequately achieved. Anto was not able to interpret the graph theory using all aspects.

The time and distance travelled graph (grade six)

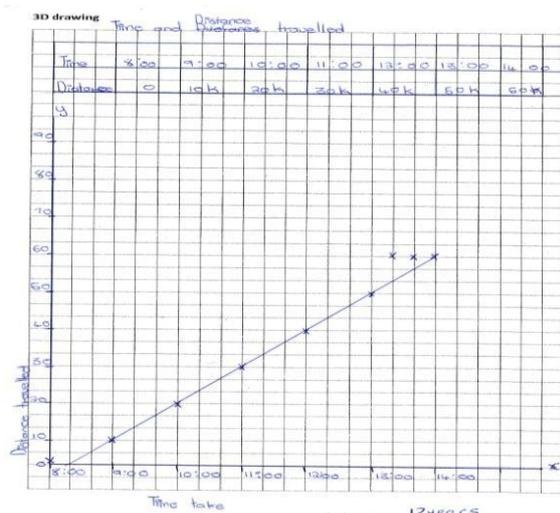
The grade six learners were given a story as follows: A car travelled every hour without stopping from 8.00 to 15 00 hours. At 8.00 it was at 000hours, 9.00 it was on 10km, 11.00 it was on 30km, at 12 00, it was at 40km, at 1300 it was at 50km, at 1400, it was at 60km and at 1600 it was 70km. Draw a table of information and a linear graph showing the travel of the car in question. Three grade six learners were interviewed using this question, these were Phee, Zwaa and Khan.

The first learner to be interviewed was Phee. Phee, a girl aged 12 was a girl from grade 6. Phee drew a table of contents showing the time, 11.12.13, 14.15 and 1600 hours. At the bottom, Phee showed the distances travelled as 0km, 10km, 20km, 30km, 40km, 50km, 60km, 60km, 70km, 80km, 90km and 100km. Considering the work presented by Phee, two aspects of the graph were shown. The vertical axis showed the distance travelled and the horizontal axis showed the time taken to travel. All the plots according to Phee were correctly placed. The graph theory was used to interpret speed, time and distance. The graph had all the aspects of the graph put in place. The following is the graph presented by Phee.

The responses of Phee



The second learner to be interviewed in this category was Zwaa. Zwaa, a girl aged 12 was a girl from grade 6. Phee drew a table of contents showing the time, 11.12.13, 14.15 and 1600 hours. At the bottom, Zwaa showed the distances travelled as 0km, 10km, 20km, 30km, 40km, 50km, 60km, 60km, 70km, 80km, 90km and 100km. Considering the work presented by Zwaa, two aspects of the graph were shown. The vertical axis showed the distance travelled and the horizontal axis showed the time taken to travel. All the plots according to Zwaa were correctly placed. The graph theory was used to interpret speed, time and distance. The graph had all the aspects of the graph put in place. The following is the graph presented by Zwaa. Below are the responses given by Zwaa:



The graph presented by Zwaa was completed by joining the points but it was not named as the straight-line graph. According to Zwaa's interpretation, the graph theory was used as a powerful tool to interpret speed, distance and time.

Grade 7(seven) responses

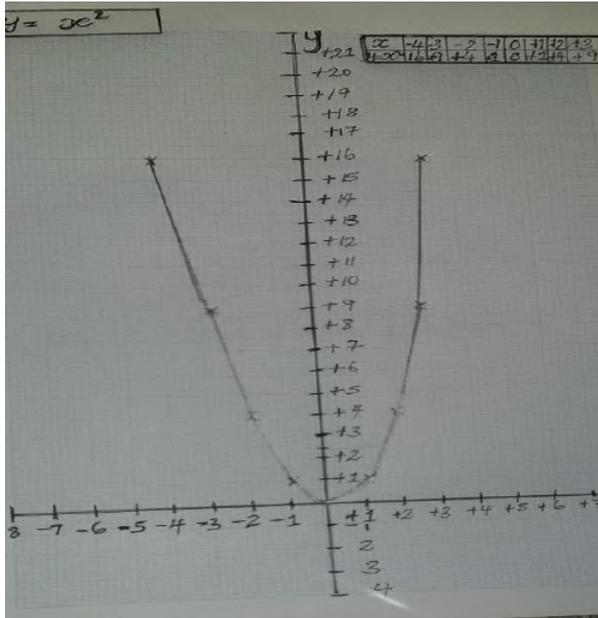
Grade seven learners were interviewed and the data were collected and analysed. The grade seven learners interviewed were Mitchie, Ty, Ally and Aides. The first grade seven learner to be interviewed was Mitchie. Mitchie a girl aged 13 is in grade 7. Mitchie was given the equation $y = x^2$. She was helped to make the table of information by the researcher. Mitchie drew the table of information for y and x .

The responses from Mitchie.

Considering the interviews held with Mitchie, the graph theory showed the values of x as from -4 to +4 and these values were well calculated by Mitchie. Mitchie could calculate the squares of given numbers.

| | | | | | | | | | |
|-----|-----|----|----|----|---|----|----|----|-----|
| x | -4 | -3 | -2 | -1 | 0 | +1 | +2 | +3 | +4 |
| y | +16 | +9 | +4 | +1 | 0 | +1 | +4 | +9 | +16 |

| | |
|---|--|
| $y = x^2$ $y = -4x - 4$ $y = +16$ | $y = x^2$ $y = -3x - 3$ $y = +9$ |
| $y = x^2$ $y = -2x - 2$ $y = +4$ | $y = x^2$ $y = -1x - 1$ $y = +1$ |
| $y = x^2$ $y = -0x - 0$ $y = 0$ | $y = x^2$ $y = +1x + 1$ $y = +1$ |
| $y = x^2$ $y = +2x + 2$ $y = +4$ | $y = x^2$ $y = +3x + 3$ $y = +9$ |
| $y = x^2$ $y = +4x + 4$ $y = +16$ | |



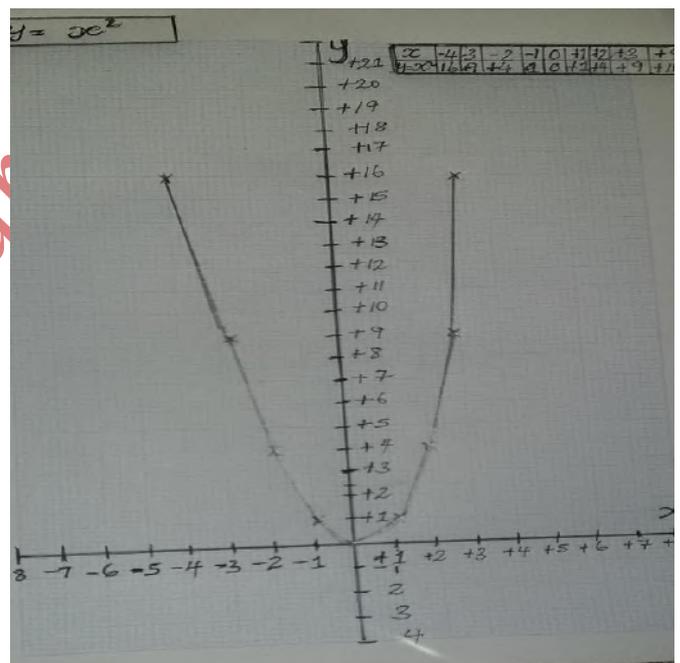
| | |
|---|--|
| $y = x^2$ $y = -4x - 4$ $y = +16$ | $y = x^2$ $y = -3x - 3$ $y = +9$ |
| $y = x^2$ $y = -2x - 2$ $y = +4$ | $y = x^2$ $y = -1x - 1$ $y = +1$ |
| $y = x^2$ $y = -0x - 0$ $y = 0$ | $y = x^2$ $y = +1x + 1$ $y = +1$ |
| $y = x^2$ $y = +2x + 2$ $y = +4$ | $y = x^2$ $y = +3x + 3$ $y = +9$ |
| $y = x^2$ $y = +4x + 4$ $y = +16$ | |

The graph theory was correctly used to interpret the equation $y = x^2$, the y-axis was correctly positioned, the x-axis was correctly positioned and the coordinates were placed in the correct position. The graph interpretation was consistent and the graph theory was used as a tool to interpret the concept of equations and squares of numbers together with the squaring of negative numbers.

The responses from Ty

Considering the interviews held with Ty, the graph theory showed the values of x as from -4 to +4 and these values were well calculated by Ty, Ty could calculate the squares of given numbers.

| | | | | | | | | | |
|-----|-----|----|----|----|---|----|----|----|-----|
| x | -4 | -3 | -2 | -1 | 0 | +1 | +2 | +3 | +4 |
| y | +16 | +9 | +4 | +1 | 0 | +1 | +4 | +9 | +16 |



The graph theory was correctly used to interpret the equation $y = x^2$, the y-axis was correctly positioned, the x-axis was correctly positioned and the coordinates were placed in the correct position. The graph interpretation was consistent and the graph theory was used as a tool to interpret the concept of

equations and squares of numbers together with the squaring of negative numbers.

The responses from Aids

Considering the interviews held with Aids, the graph theory showed the values of x as from -4 to +4 and these values were well calculated by Aids, Aids, could calculate the squares of given numbers.

| | | | | | | | | | |
|-----|-----|----|----|----|---|----|----|----|-----|
| x | -4 | -3 | -2 | -1 | 0 | +1 | +2 | +3 | +4 |
| y | +16 | +9 | +4 | +1 | 0 | +1 | +4 | +9 | +16 |

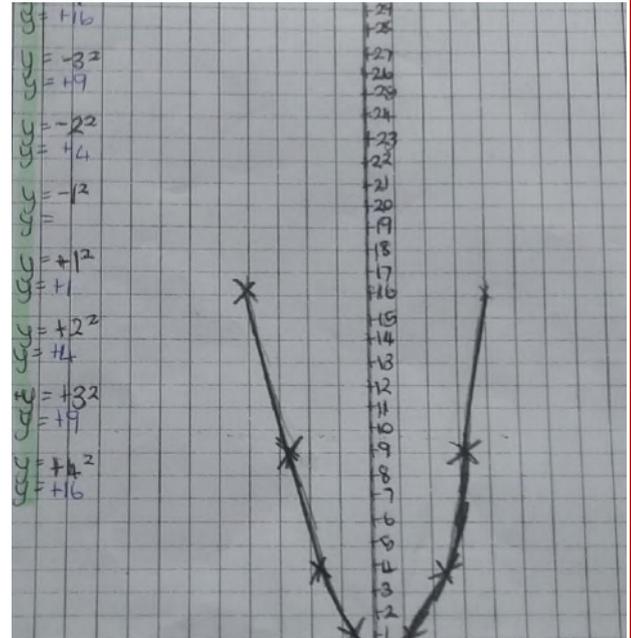
| | |
|---|--|
| $y = x^2$ $y = -4x - 4$ $y = +16$ | $y = x^2$ $y = -3x - 3$ $y = +9$ |
| $y = x^2$ $y = -2x - 2$ $y = +4$ | $y = x^2$ $y = -1x - 1$ $y = +1$ |
| $y = x^2$ $y = -0x 0$ $y = 0$ | $y = x^2$ $y = +1x + 1$ $y = +1$ |
| $y = x^2$ $y = +2x + 2$ $y = +4$ | $y = x^2$ $y = +3x + 3$ $y = +9$ |
| $y = x^2$ $y = +4x + 4$ $y = +16$ | |

The graph theory was correctly used to interpret the equation $y = x^2$, the y -axis was correctly positioned, the x -axis was correctly positioned and the coordinates were placed in the correct position. The graph interpretation was consistent and the graph theory was used as a tool to interpret the concept of equations and squares of numbers together with the squaring of negative numbers.

The graph theory

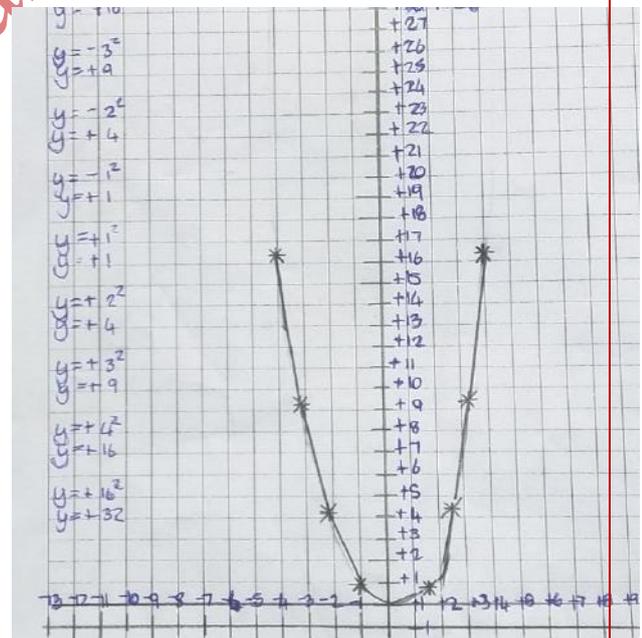
The next section is an analysis of the work from a Grade 7 learner. Ally a girl aged 13 and was asked to draw up a graph and write questions and answer questions concerning the graph theory. Considering the responses provided by Ally she could draw the

vertical and the horizontal axis. Below is the graph and calculation gave by Ally.

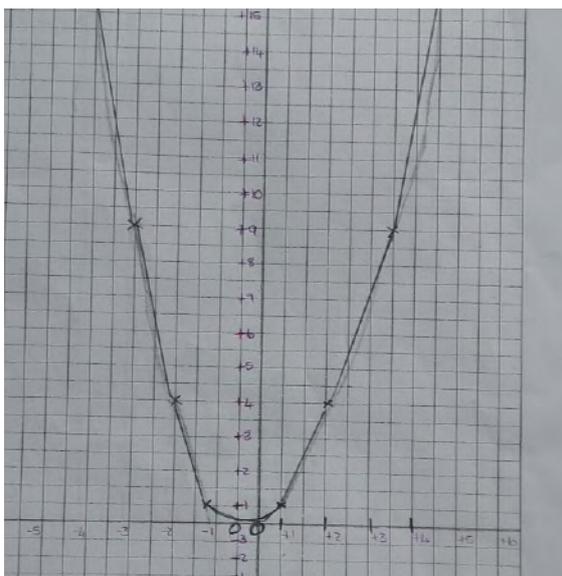


According to the responses were given by Ally all calculations were shown vertically.

The following are the responses given by Aids



The following is a graph produced by Ty



Findings

This article has advantages and disadvantages. The following advantages were picked in this article considering the responses of the grade one to grade seven learners as the advantages:

- The graph theory can be taught to all learners from pre-school to grade seven.
- The graph theory enhances several other mathematical skills such as sorting, ordering, counting, drawing, recording and analyzing.
- Graph theory can be a powerful tool used to teach mathematical concepts.
- A graph is worth a million words in terms of explanation and analysis.
- New concepts can be learnt using graph theory and old concepts can be enhanced.
- The graph theory can be a connection thread for all grades from pre-school to the University as long as the teacher/facilitator uses the procedures from the simple to the complex.
- Graphs and charts were used to condense large amounts of information into easy-to-understand formats that clearly and

effectively communicate important points at all levels in this article.

This article points out the disadvantages of using the graph theory at an early stage. The disadvantages of using the graph theory are as follows:

- There is too much detail in the graph that not even one learner could include all the required data on the graphs.
- Those learners find it difficult to write full questions and full answers when working with graphs.
- Some graphs may have complicated information for example the equation $y = x^2$ in grade seven appear to be a difficulty for the learners.
- Learners with some language barriers may not be willing to learn this way using the graph theory because they fear some of their hidden challenges such as spelling, inaccuracy will be picked up.

Conclusion

This article concludes that the graph being a concept, a task or a theory can be a powerful tool used to build other mathematical concepts if it is properly and carefully executed.

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Author Profile

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Pardon Marufu received a Certificate of Education (1989) from the University of Zimbabwe and B-Tech in Education Management (2002). The researcher has Honours in Education Management with the University of South Africa in (2017). During 1989-2008, he stayed in the Ministry of Education of Zimbabwe teaching all primary school subjects. From 2009 to 2021, he is teaching Mathematics, Natural Science and Technology in South African Schools. Marufu Pardon completed Masters in Education in 2020.

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