

# Optimization of Production Planning Using Linear Programming

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## ABSTRACT

*The production process carried out by an industry aims to meet customer demands accurately and quickly through a production process. If the production results are not following customer demand, whether it is a shortage or excess product, it can result in increased costs incurred by the industry. Therefore, it is necessary to carry out a comprehensive production planning (Aggregate Production Planning) by considering the number of customer requests and the resources or capacity owned by the industry. The purpose of this research is to find a comprehensive optimization system model of the production planning, to optimize the production costs incurred and the level of profit. The methods used are the forecasting method, the Aggregate Production Planning method, and linear programming. The variables and parameters used are the appropriate production factors to obtain an optimization model for aggregate production planning. The result of this research is an optimization model of aggregate production planning using linear programming, which is obtained through the integration of linear programming model and aggregate production planning model, with decision variables and parameters covering various production factors to attain a minimum total cost.*

**Keywords: Aggregate Production Planning, Linear Programming, Optimization, Integration**

## 1. INTRODUCTION

An industry must carry out a production process, to meet customer demands accurately and quickly both in terms of product quantity and quality [1]. If

the amount of production does not match demand, there can be a shortage of products or excess products (stock), which can result in increased inventory costs [2]. To achieve this goal, it is necessary to carry out comprehensive production planning, considering the number of customer requests and the resources or capacity owned by the company. As a first step before carrying out the planning, customer demand data is needed, including the calculation of customer demand for the future period, so that production planning is based on the estimated number of requests [3].

Production planning is planning about the type of product and the amount to be produced in the future period. What needs to be considered in production planning is the achievement of optimization of profits and costs [4]. While the purpose of aggregate planning is to meet customer demand and minimize costs in the planning period. Planning is carried out to meet total needs by using all resources (human and equipment) contained in the company [2]. A plan that is not carried out systematically will have an impact on unfulfilled customer demand due to a shortage of products, which can lead to reduced company profits, or product accumulation in warehouses due to excess products, and resulting in increased inventory costs [4]. For this reason, research is needed to solve this problem.

In line with this description, a comprehensive production plan is needed by considering various factors in production, and through integrating various methods in linear programming.

Consequently, optimization of production costs and profits will be obtained by the company.

The methods used are generally separate calculations between the use of formulas contained in the aggregate and modeling in linear programs, each of which is used to make production plans. In some studies, linear programming has been used for the calculation of aggregate planning. This study includes the aggregate planning obtained by using the formulas in the aggregate planning method and obtained from solving the linear programming model using the variables commonly found in linear programming. From the analysis results, a linear programming model with decision variables and parameters will be obtained which is more suitable for aggregate planning. Thus, in this study, a flow of calculations will be obtained to get a linear program model that is used to design an aggregate plan.

## 2. MATERIAL AND METHODS

### 2.1. Production Planning

Production planning is an activity to produce a product according to the needs of the company and customers [5], that is, determining a production plan with minimal costs or maximum profit considering that demand must be met when capacity is available. Demand for each period is a combination of customer orders with the same due date [6]. The fulfillment of customer demand is adjusted to the capacity/resources owned. If the resources are limited, it is necessary to make a priority scale to determine which customer orders are fulfilled. For this reason, it is necessary to carry out integrated production planning. Production planning methods that iterate between planning and scheduling levels to determine the correct values for the parameters of the production planning model have been developed for various production planning problems [7]. Production planning is influenced by the complexity of variables including, the number of products, processes and processing units, storage constraints related to time and quantity, and non-productive, activities that depend on flexible sequences and crews [8].

### 2.2. Forecasting

Forecasting is a process for predicting future events through the calculation of previously processed data systematically based on a predetermined calculation process [9].

### 2.3. Linier Programming

Linear programming is a mathematical technique to determine the best allocation of a company's limited resources to achieve optimal goals. It is one of the most widely used optimization techniques and perhaps the most effective method [10]. Linear Programming is an optimization technique for allocating limited resources (covering all production factors, such as machinery, labor, raw materials, capital, available technology), to optimize the objective function (maximizing or minimizing) on a function. the linear objective with equations and linear inequality constraint functions [11]. The problem of resource allocation will arise if there are certain activities that require meeting needs. These problems will be translated into mathematical formulations [12]. In addition, the linear programming technique is an optimization model of resource allocation to achieve efficiency in production planning [13]. Linear programming can be applied to create a production schedule that aims to optimize resources. Linear programming is an operational research technique that is widely used in various management problems [14]. It is a mathematical model that is used to solve problems regarding the determination of several things [15]:

- a. The amount of input data used in a problem.
- b. Combination of available input data or combination of output data to be generated.
- c. The amount of output data produced to achieve the optimization objective of a case, for example, to achieve maximum profit or minimum cost of capital.

The linear programming model, in general, is [16]:

Optimize (max or min)  $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

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$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n (\leq, =, \geq) b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n (\leq, =, \geq) b_2$

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$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n (\leq, =, \geq) b_m$

The model can also be expressed in the following form:

Optimize (max or min)  $Z = \sum_{j=1}^n c_j x_j$  (Objective Function)

Linear Constraints:

$$\sum_{j=1}^n a_{ij} x_j (\leq, =, \geq) b_i, i = 1, 2, \dots, m \text{ and}$$

$$x_j \geq 0, j = 1, 2, \dots, n$$

$c_1, c_2 \dots c_n$  = cost per unit of the decision variable  
 $x_1, x_2 \dots x_n$

$a_{11}, a_{12}, \dots, a_{2n}, \dots, a_{m1}, \dots, a_{mn}$  = amount of material consumed per unit of the decision variable

$b_i$  = total jumlah kapasitas

$x_i$  = variabel keputusan atau variabel input

$Z$  = ukuran kinerja dapat berupa keuntungan atau biaya

For product demand constraints [17]:

The sum of each product produced in different lines must meet the demand for that type of product, which is formulated as follows:

$$\sum_{t=1}^{Nt} \sum_{l=1}^{Nl} \beta_{qlt} Q_{ql} \geq D_q$$

$Q_{ql} = D_q$  = the parameters of each that determine the quantity of product  $q$  units produced in line  $l$  and the total demand for product  $q$ .

$q_{lt}$  = binary variable representing the state of manufacture of a product of type  $q$ , which is equal to 1 if produced in row  $l$  at time  $t$ , and 0 otherwise.

The non-negative condition is  $x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0$  [18].

## 2.4. Aggregate Planning

Aggregate planning is medium-term capacity planning that determines the minimum cost, labor, and production plans required to meet customer demands. Aggregate planning, simultaneously establishing optimal production, inventory, and work levels over a finite planning period satisfies the total demand for all products that share the same limited resources [19]. Aggregate production planning is intended to balance production capacity and quantity requirements for the medium-term planning horizon. As it provides the basic input for the next planning step [20], aggregate planning is a very important part of operations management [21]. Aggregate planning inputs are the amount of demand and capacity. Variables in aggregate

planning are the level of inventory, the amount of monthly production and labor, and the level of subcontracting [22].

## 3. SIMULATION CASE

This research shows how is the process of making a linear programming model gets the optimization of aggregate planning.

### 3.1. Linier Programming

In this step, the creation of a mathematical model is based on variables that are commonly considered in linear programming. Since in this study the objective function is the minimum cost, with the constraints of demand for the future period, the capacity owned by the company consisting of the number of hours worked and raw materials, the notation is made as follows.

$X_i$  = Number of Products Produced Per  $i$  Period

$U_j$  = Regular Production Cost

$K_{Pi}$  = Production Capacity of  $i$  Period

$D$  = Total Demand for the next 12 Periods

$K_{Ba}$  = A Raw Material Requirement

$K_{BBa}$  = A Raw Material Capacity

$K_{Bb}$  = B Raw Material Needs

$K_{BBb}$  = B Raw Material Capacity

$K_{Bc}$  = C Raw Material Needs

$K_{BBc}$  = C Raw Material Capacity

$W_i$  = Time to Make Product  $i$  Period

$K_{tki}$  = Labor Hour Capacity

Under these conditions:

1 day = 8 hours of work

1 month = 26 working days

1 year = 12 months

Model Development:

$$Z_{min} = \sum_{i=1}^{i=12} X_i \times U_j$$

Constraint:

Production capacity:  $X_i \leq K_{Pi}$

Demand :  $\sum_{i=1}^{i=12} X_i \geq D$

Raw material:

A:  $K_{Ba} \times X_i \leq K_{BBa}$

B:  $K_{Bb} \times X_i \leq K_{BBb}$

C:  $K_{Bc} \times X_i \leq K_{BBc}$

Labor :  $W_i \times X_i \leq K_{tki}$

$X_1, X_2, \dots, X_{12} \geq 0$

As a simulation, the linear programming model is as follows:

1. Purpose Function:

$$Z_{min} = 20,000 X_1 + 20,000 X_2 + \dots + 20,000 X_{12}$$

2. Constraints:

Demand Constraints:

$$X_1 + X_2 + \dots + X_{12} = 2217$$

Production Capacity Constraints:

$$X_1 \leq 185; X_2 \leq 185; X_3 \leq 185; X_4 \leq 185; X_5 \leq 185; X_6 \leq 185; X_7 \leq 185; X_8 \leq 185; X_9 \leq 185; X_{10} \leq 185; X_{11} \leq 185; X_{12} \leq 185$$

A Raw Material Capacity Constraints:

$$25 X_1 \leq 7200; 25 X_2 \leq 7200; 25 X_3 \leq 7200; 25 X_4 \leq 7200; 25 X_5 \leq 7200; 25 X_6 \leq 7200; 25 X_7 \leq 7200; 25 X_8 \leq 7200; 25 X_9 \leq 7200; 25 X_{10} \leq 7200; 25 X_{11} \leq 7200; 25 X_{12} \leq 7200$$

B Raw Material Capacity Constraints:

$$2 X_1 \leq 720; 2 X_2 \leq 720; 2 X_3 \leq 720; 2 X_4 \leq 720; 2 X_5 \leq 720; 2 X_6 \leq 720; 2 X_7 \leq 720; 2 X_8 \leq 720; 2 X_9 \leq 720; 2 X_{10} \leq 720; 2 X_{11} \leq 720; 2 X_{12} \leq 720$$

C Raw Material Capacity Constraint:

$$7 X_1 \leq 2160; 7 X_2 \leq 2160; 7 X_3 \leq 2160; 7 X_4 \leq 2160; 7 X_5 \leq 2160; 7 X_6 \leq 2160; 7 X_7 \leq 2160; 7 X_8 \leq 2160; 7 X_9 \leq 2160; 7 X_{10} \leq 2160; 7 X_{11} \leq 2160; 7 X_{12} \leq 2160$$

Kendala Jam Tenaga Kerja:

$$2 X_1 \leq 2490; 2 X_2 \leq 2490; 2 X_3 \leq 2490; 2 X_4 \leq 2490; 2 X_5 \leq 2490; 2 X_6 \leq 2490; 2 X_7 \leq 2490; 2 X_8 \leq 2490; 2 X_9 \leq 2490; 2 X_{10} \leq 2490; 2 X_{11} \leq 2490; 2 X_{12} \leq 2490$$

Syarat:

$$X_1, X_2, \dots, X_{12} \geq 0$$

The model is then processed using POM software and the results show that from period 1 to 11, the number of products produced is 185,000 boxes, while in period 12 it is 182,000 boxes with a total cost of IDR. 44,340,000,000, -

### 3.2. Aggregate Planning

At this stage, the calculation was carried out using the steps contained in the aggregate planning method. By using the data of regular production capacity of 185,000 was doses, subcontract was 10,000 doses, and overtime was 25,000 doses. Regular production costs IDR. 20,000,000. -, subcontract fee was IDR. 8.525.000, -; and the overtime fee was IDR. 7,567,567. -. Inventory cost was IDR. 2,250,000. -, shortage fee was IDR. 2.535.000, -, increase fee was IDR. 1,515,000, -, and the cost decreased by IDR 1,515,000, -.

Calculations were carried out using POM software, with the following results:

**Table 1. Aggregate Calculation Output**

Period	Regular Time Production	Inventory	Unit Decrease
1	185	1	0
2	185	2	0
3	185	3	0
4	185	4	0
5	185	5	1
6	185	5	0
7	185	5	0
8	185	5	0
9	185	5	0
10	184	4	1
11	184	2	0
12	184	0	0
<b>Total (units)</b>	2217	49	1
<b>Total Cost</b>	IDR. 44,433,760,000, -		

Table 1 shows the number of products produced per period (regular time units) and considers the number of inventory and decrease units.

This research integrates forecasting methods, aggregate planning, and linear programming. Thus, a calculation process flow was obtained, resulted in a production planning optimization system model. Began with the use of forecasting methods to obtain predictions of future demand, then was processed using aggregate methods and linear programming, together with the other data. The result of both methods is the number of products that must be produced each month, with different output details.

The use of the aggregate method will show the quantity produced regularly, overtime, subcontract, the amount of inventory, the number of shortages, the number of decreases and the number of increases, and finally the total optimal cost incurred is listed. Meanwhile, with a linear program, the result is the number of products that must be produced each month and the optimal total cost incurred. For this reason, an optimization model that combines the aggregate method and linear programming, to obtain detailed results using the desired decision parameters and variables, is made.

### 3.3. Linear Program Model for Production Planning Optimization

At this stage, based on the results of the two previous methods, a mathematical model is obtained that describes a linear programming model for optimization of production planning, with the desired parameters and decision variables. The mathematical model is as follows:

Parameters used:

- $D_i$  = Forecasting future customer demand
- $RTC_i$  = Regular Time Capacity of  $i$  period
- $OTC_i$  = Overtime Capacity of  $i$  period
- $SC_i$  = Subcontract Capacity of  $i$  period
- $CR$  = Regular production cost
- $CO$  = Overtime production cost
- $CS$  = Subcontract production cost
- $CI$  = Inventory cost
- $CSH$  = Shortage fee
- $C_{in}$  = Cost increase
- $C_{de}$  = Cost decrease
- $i$  = Period 1, 2, ....

The decision variables in this study are:

- $XiR$  = Total production of regular time of  $i$  period
- $XiO$  = Total overtime production of  $i$  period
- $XiS$  = Total production of sub-contract of  $i$  period
- $I_i$  = Ending product inventory level for  $i$  period
- $L_{bi}$  = Number of workers in period  $i$
- $L_{in}$  = Number of workers recruited in  $i$  period
- $L_{ide}$  = Number of workers released in  $i$  period
- $SH_i$  = Number of product shortages in  $i$  period
- $Z_{tot}$  = Total costs incurred

Modeling:

Purpose Function:

$$Z_{min} = CR XiR + CO XiO + CS XiS + CI I_i + CSH SH_i + C_{in} L_{in} + C_{de} L_{ide}$$

Constraints:

Demand Constraints:

$$XiR + XiO + XiS + I_i - I_{i-1} = D_i$$

Regular Capacity Constraints:

$$XiR \leq RTC_i$$

Overtime Capacity Constraints:

$$XiO \leq OTC_i$$

Subcontract Capacity Constraints:

$$XiS \leq SC_i$$

Barriers to Recruitment and Dispatch of

Manpower:

$$L_{bi} - L_{bi-1} = L_{in} - L_{ide}$$

$$XiR \geq 0; XiO \geq 0; XiS \geq 0; I_i \geq 0; L_{bi} \geq 0; L_{in} \geq 0; L_{ide} \geq 0; SH_i \geq 0$$

### 3. CONCLUSION

Optimization of production planning in aggregate by using a linear program is more flexible since it can include variables that are more suitable for the company's conditions. In this study, there is a sequential data processing flow, so that the difference between the use of formulas in the aggregate planning method and the completion of a linear programming model using commonly used variables can be seen clearly. The results of the analysis on the use of these two methods produce an optimization model for aggregate production planning, with decision variables and parameters that have been adjusted to the company's conditions to obtain a comprehensive aggregate plan.

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