

Discussions on Two Major Problems in Computation and Contradiction

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ABSTRACT

Russell's paradox shows that the set of all sets does not exist, which gives an example of set theoretical paradox. To resolve a version of this paradox, Russell proposed a theory of types, and then the type hierarchy to infinity appeared in the resolution of the paradox. Victor gave a solution to remove the infinite type hierarchy by proposing his Duality Principle, which not only can solve Russell's paradox, but also can solve the Liar's paradox. In this paper, I contend the completeness of meanings in Duality Principle. Firstly, in solving the Russell's paradox, I assert that the complete meaning of a set should contain Miscellaneous Sets such as $A=\{a,\{b\}\}$, in addition to Pure Sets $\{a,b,c\}$ and Set of Sets $\{\{a\},\{\{b\},c\}\}$. And this will require solution of Russell's paradox by Duality principle change its disjunction condition to have 3 conditions. i.e A set is either a Pure Set, or a Set of Sets, or a Miscellaneous Set. Then his original solution can not follow these new conditions of the Duality Principle. Secondly, in resolving the Liar's paradox by Victor's Duality Principle, I argue that the

completeness of meaning of a statement, in addition to {a statement, a statement about statements}, there should be also { a statement about (statement about statements) }. It means that the original meaning of a statement is incomplete, there are more meanings of a statement such that it is a complete set of meanings. However, this will push us to Russell's infinite type hierarchy which Victor tried to avoid. Therefore, I myself would like to give the meaning of a statement in Liar's paradox as a Boolean function, which in turn renders the Dialetheism.

Keywords: *Computation, Contradiction, Duality, Liar, Paradox, Russell*

1. INTRODUCTION TO RUSSELL'S PARADOX [1]

Let a set A be defined by the following sentence

$$A=\{x : x \text{ is a set, } x \text{ does not belong to } x \}$$

Thus if A belongs to A, then A does not belong to A because x does not belong to x with replacing x by A.

So this means that A belongs to A is false.

And if A does not belong to A, then because x does not belong to x with replacing x by A, A belong to A. So this means that A does not belong to A is false.

Both "A belongs to A" and "A does not belong to A" are false, so we meet a contradiction here. Thus A does not exist. This means that a formally constructed set could be non-existent.

2. INTRODUCTION TO LIAR'S PARADOX[2]

I am lying.

If I am lying, then the sentence "I am lying" should be false. Hence I am not lying, a contradiction.

If I am not lying, then what I speak is true, so "I am lying" is true, which contradicts with the premise "I am not lying". Therefore we get a contradiction again.

Thus in each direction, we got contradictions. So the proposition function is neither true nor false.

Some other versions of Liar's paradox are listed below.

2.1 $L : L$ is false.

If L is true, then L is false. And if L is false, then L is true. Similarly in each direction we get a contradiction. In each way we got a contradiction. Hence L is neither true nor false.

2.2 The following sentence is true.

The above sentence is false.

If the following sentence is true, then the above sentence is false, which means the following sentence is false, a contradiction. Similarly, if the following sentence is false, then the above sentence is true, which means the following sentence is true, a contradiction with the premise.

In each way we got a contradiction. Hence the sentence "The following sentence is true" is neither true nor false.

3. INTRODUCTION TO DUALITY PRINCIPLE [7]

Before introducing Victor's Duality principle, we must firstly introduce the Simultaneity Constraint [4] :

"These observations from the world of our experiences form the basis of the principle that we will refer to as the Simultaneity Constraint: The mind cannot make more than one judgement at the same time."

And we can state the Simultaneity Constraint in another form [5]

"Sentences are statements about objects. This means that the Simultaneity Constraint can be made more specific by referring to the predicates of an object. We can in fact state the Simultaneity Constraint as:

An object cannot simultaneously possess two distinct predicates."

Then we can now introduce the duality principle [6]

"And now we can apply the Simultaneity Constraint to the grammatical particles of conjunction and disjunction themselves. This yields the following important principle that we refer to as the Duality Principle:

we cannot compound statements by conjunction and disjunction simultaneously"

And we can restate the duality principle in the following form [7]

"We can therefore state the Duality Principle as:

x is P and x is Q implies that it is not the case that x is P or x is Q . x is P or x is Q implies that it is not the case that x is P and x is Q "

This is the end of the introduction of Duality Principle.

4. SOLUTION OF RUSSELL'S PARADOX BY DUALITY PRINCIPLE [8]

"In set theory, following the example of Russell, we can define two types: 'sets' and 'sets of sets'. A set is an object that can be a member of a set of sets while a set of sets is an object that can have sets as

members.

We invoke the Duality Principle to stipulate that for any object x , within scope:

Conjunctive Duality Principle:

x is a set and x is a set of sets implies that it is not the case that x is a set or x is a set of sets

Disjunctive Duality Principle:

x is a set or x is a set of sets implies that it is not the case that x is a set and x is a set of sets

Assume the disjunctive Duality Principle is true. Then this implies that "x is a set or x is a set of sets" is true within scope. Then Russell's Paradox can be analyzed as follows:

Either Z is a set or Z is a set of sets.

If Z is a set then Z is not a set of sets. Z cannot have sets as members, itself included.

If Z is a set of sets then Z is not a set. Z cannot be a member of any set of sets, itself included.

In either case there is no contradiction. Thus using the disjunctive Duality Principle together with a type duality of the set concept resolves Russell's Paradox."

5. SOLUTION OF LIAR'S PARADOX BY DUALITY PRINCIPLE [9]

"Let us represent the statement in the Liar Paradox as y : y is false. Let us define two dual types: statements and statements about statements. Invoking the Duality Principle we have:

Conjunctive Duality Principle:

x is a statement and x is a statement about statements implies that it is not the case that x is a statement or x is a statement about statements

Disjunctive Duality Principle:

x is a statement or x is a statement about statements implies that it is not the case that x is a statement and x is a statement about statements

Assume the disjunctive Duality Principle is true. Is y a statement or a statement about statements?

If y is a statement then it is not a statement about statements; y is a meaningless statement since y cannot reference statements. If y is a statement about statements then there y is again meaningless since statements of statements cannot reference statements of statements."

6. DISCUSSION OF SOLUTION OF RUSSELL'S PARADOX BY DUALITY PRINCIPLE

Here I contend that the use of set and set of sets in the proof does not complete the meaning of "set".

I give the my definition of complete set meanings below,

A "Set" can be one of the following types of set

1. Pure Set : which contains only individuals : example $\{a,b,c,d\}$
2. Set of Sets : which contains only "Sets" : example : $\{\{a\},\{b\},\{\{c\},d\}\}$
3. Miscellaneous Set : which contains both individuals and "Sets" $\{a,\{b\}\}$
4. Empty Set

However, Victor's Disjunctive Duality Principle says "Either Z is a set or Z is a set of sets." [8] Firstly, it misses the cases of Miscellaneous Set, so the meaning of set is incomplete. Secondly, the "set of sets" used by Victor only refers to "set of Pure Set". This is very much different from my definition of 2. Set of Sets, which refers to "set of Set".

Therefore in the solution of Russell's paradox, it says

"If Z is a set of sets then Z is not a set. Z cannot be a member of any set of sets." [8] is not true by my definition of set, which allows Z be a member of some set of Sets. For example, Z is a set of Sets $\{\{a\},\{b\}\}$, it can be a member of set of Sets $\{\{\{a\},\{b\}\},\{\{c\},\{d\}\}\}$.

As a result, Victor's Disjunctive Duality Principle which says "Either Z is a set or Z is a set of sets." [8] should be replaced by "Either Z is a Pure Set, or Z is

a Set of Sets, or Z is a Miscellaneous Set". Firstly, this completes the meaning of a "Set". Secondly, this means the resolution by Duality Principle is problematic with the new definition of Set.

7. DISCUSSION OF SOLUTION OF LIAR'S PARADOX BY DUALITY PRINCIPLE

In the solution of Liar's paradox by Duality principle, Disjunctive Duality Principle says

x is a statement or x is a statement about statements

I argue that this is also an incomplete meaning of statement. Because x could also be "a statement about (statements about statements)", which is not listed in the Disjunction Duality Principle.

Hence, the following is not true with the addition of "a statement about (statement about statements)".

"If y is a statement then it is not a statement about statements; y is a meaningless statement since y cannot reference statements. If y is a statement about statements then there y is again meaningless since statements of statements cannot reference statements of statements." [9]

Because y is now meaningful since "a statement about (statements about statements)" can reference statements of statements.

Is this a complete meaning of statements? Yet it is

still not complete because we still can have "a statement about (statements about (statements about statements))". So again we get a type hierarchy to infinity, though it actually completes the meaning of statement.

To avoid the type hierarchy and incompleteness of meanings. I would myself suggest treating y as a Boolean function with reference to itself.

$y : y$ is false

Therefore this solution avoids the type hierarchy and incompleteness of meanings. However it does not solve the paradox. Indeed this gets me back to the Dialetheism [10], which is the view that there are statements which are neither true nor false.

Lastly, one fallacy by inspecting the solution of Liar's paradox by Duality principle is that if we change the Liar's paradox's content to y is true, i.e.

$\implies y : y$ is true.

The argument of the Duality principle still remains, but now there is apparently no paradox to be solved..

8. CONCLUSION

I have shown you the two famous paradoxes : Russell's paradox and Liar's paradox, one is a set theoretical paradox while the other is a language paradox. Then I introduce Victor's Duality Principle. And what follows is the solutions of Russell's

paradox and Liar's paradox by Victor's Duality principle. Next in this paper, I contend the completeness of meanings in Duality Principle. Firstly, in resolving the Russell's paradox, I assert that the complete meaning of a set should contain Miscellaneous Sets. And this will require solution of Russel's paradox by Duality principle change its disjunction condition to have 3 conditions. Secondly, in resolving the Liar's paradox by Victor's Duality Principle, I argue that the completeness of meaning of a statement, in addition to {a statement, a statement about statements}, there should be also { a statement about (statement about statements) }. However, this will push us to Russell's infinite type hierarchy which Victor tried to avoid. Therefore, I myself would like to give the meaning of a statement in Liar's paradox as a Boolean function, which in turn renders the Dialetheism.

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[7] Duality Principle

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[8] Solution of Russell's Paradox by Duality Principle

[3]pp164

[9] Solution of Liar's Paradox by Duality Principle

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[10] Dialetheism

<https://en.wikipedia.org/wiki/Dialetheism>

9. REFERENCES

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[4] Simultaneity Constraint

[3]pp128

[5] Simultaneity Constraint

[3]pp129

[6] Duality Principle