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Stochastic Finite Automata: A Mathematical Model for Optimal Medical Diagnosis under Uncertainty

Author: Kulandai Therese. A; James Albert. A

Affiliation: Ph.D scholar, Karpagam University and Associate Professor of Mathematics, Nirmala College for Women, Coimbatore, India; Professor and Head, Dept. of Science and Humanities, Hindusthan College of Engineering and Technology, Coimbatore, Tamil Nadu, India

E-mail: infanta1960@gmail.com; asiralbert@gmail.com

ABSTRACT

Human body is very complicated and it is characterized by many diseases. Diseases and its diagnosis are greatly influenced by one's body constitution, symptoms and knowledge of the diagnostician (physician). An experienced physician with his knowledge would be in a better position to make correct decisions or correct diagnosis. The process of Medical Diagnosis is often difficult because of the simultaneous presence of multiple disorders and the possibility of a disorder appearing without its complete manifestations.

Probabilistic Transition System (PTS) is an extension of Labeled Transition System where each transition depends on a probability. PTS constitutes a framework for the description and comparison of processes with stochastic behaviour. Stochastic Finite Automata is suitable for the construction of mathematical models of complex systems having stochastic behavior in a finite way. In the study, PTS is extended by adding a component so as to represent a Transition System for Sequential Decision Problem under Uncertainty. The study is an attempt to showcase Stochastic Finite Automata as a Mathematical Model for Optimal Medical Diagnosis under Uncertainty

Keywords

Medical Diagnosis, Symptoms, Diseases, Physician, Uncertainty, Probabilistic Distribution, Probabilistic Transition System, Stochastic Finite Automata

1. Introduction

Human body is very complicated and it is characterized by many diseases. Diseases and its diagnosis are greatly influenced by one's body constitution, symptoms and knowledge of the diagnostician (physician). Medicine as a discipline is science and not an art like painting. Neither is it a science like physics. It is an applied science. Since patients are not all identical, it can be very tricky to decide how to apply the science to different individuals. Medical diagnosis, nowadays, is not a science but rather an art of a few highly qualified professionals [11, 12]. A physician, with the right information at his disposal at the right time, would be in a better position to arrive at correct decisions or diagnose it correctly.

The objective of the study is to develop a Medical-Decision Support System (MDSS) which is combined with knowledge representation using Finite Automata with the probabilistic approach [17]. Though it is not intended as a descriptive model for decision making process of physicians, it can serve as a diagnostic decision support system. Mathematical models help in generating and clarifying hypotheses, assessing quantitative conjectures and finding answers to specific questions. Decision making in the field of medicine is complex because the symptoms of the patient is dubious in nature. A physician must be careful to apply his knowledge from a large array of entities like disease presentations, diagnostic parameters, drug combinations and guidelines [8].

Physicians are often faced with difficult choices, because of imperfect clinical data and uncertain outcomes of treatment. Probabilistic medical reasoning, an approach based on probability theory can help physicians to deal with the uncertainty inherent in many medical decisions [3]. The importance of probability in medical decision making was noted as long ago in the year 1922. Some decisions are made on the basis of deductive reasoning, or of physiological principles. Many decisions, however, are made on the basis of knowledge that has been gained through collective experience: the physician often must rely on empirical knowledge of associations between symptoms and disease to evaluate a problem.

Overview of Clinical diagnostic Process

Diagnosis is a key feature of a physician's expertise in medical practice. In other words, medical practice is medical decision making. This is a core cognitive skill, based on both knowledge and judgment which includes the ability to interpret complex information, support the patient in understanding their condition and define circumstances when patients' symptoms could have several causes, identify and advise on appropriate treatment or preventive options, and explain and discuss the risks, benefits and uncertainties of various tests and treatments [18].However, physicians will support patients to help themselves. It involves responding to the initial presentation of illness, prioritising and synthesising information, in making a clinical assessment.

Probability Distribution

Everyday medical diagnosis contains many examples of probability. "Medical decisions based on probabilities are necessary but also perilous [16].One often uses words like probably, unlikely, certainly, or almost certainly in all conversations with patients but only rarely attach numbers to these terms. Probability is the preferred means of expressing uncertainty [16] and the application of medical knowledge is always done in the context of uncertainty. Probability is represented numerically by a number between 0 and 1. Statements with a probability of 0 are false. Statements with a probability of 1 are true. Most statements from real life fall somewhere between 0 and 1. Probability of 0.5 or 50% are just as likely to be true as false..

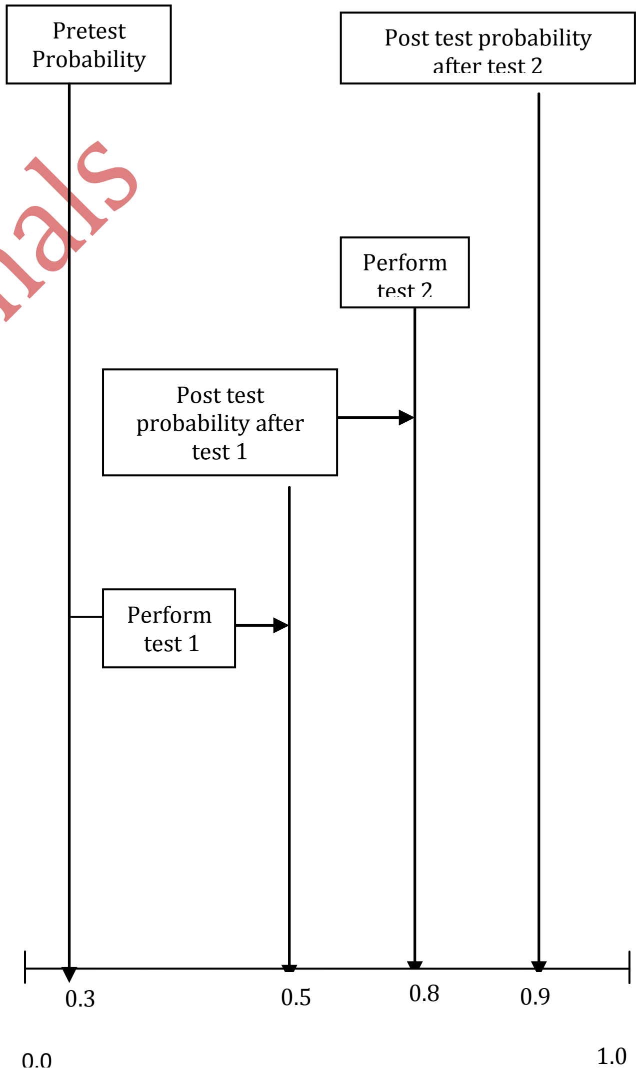
One is accustomed to accepting the fact that one's diagnoses have a certain probability of being wrong, so the patient is advised about what to do in the event ("unlikely event") that things do not work out in the expected way. Practicing and perfecting the art of medicine demands recognition that uncertainty permeates all clinical decisions [16]. Most assessments that physicians make about probability are based on personal experience. The physician may compare the current problem to similar problems encountered previously, and then ask himself, "What was the frequency of disease in similar patients whom I have already seen?" To make these subjective assessments of probability, people rely on several discrete, often unconscious mental processes that have been described and studied by cognitive psychologists. These processes are termed 'cognitive heuristics' [9, 4**]**.

One way of estimating probability is to ask themselves: What is the probability that object 'A' belongs to class 'B'? For instance, what is the probability of patients having chest pain belongs to the class of patients with Pulmonary Embolism (PE)? To answer, the physician often rely on the representativeness heuristic [1], in which probabilities are judged by the degree to which A is representative of, or similar to B. The clinician will judge the probability of the development of a PE by the degree to which the patient with chest pain resembles the clinician's mental image of patients with PE. If the patient has all the clinical findings (signs and symptoms) associated with PE, then the physician concludes that the patient is highly likely to have PE.

Another common heuristic used to judge probability is anchoring and adjustment' [10, 21]. A clinician makes an initial probability estimate known as 'pretest probability' from the symptoms and then adjusts the estimate (increase/ decrease his initial probability) based on further information (may be the history of the patient). The physician might use subjective methods to alter his estimate further based on other specific information about the patient.

Pretest and Posttest Probabilities:

While using Stochastic Finite Automata for optimal diagnosis, the physician should be aware of the 'pretest probability' of the suspected disease. The method that physician can use to make his diagnosis about the probability of disease before prescribing certain clinical tests. In this framework, probability 'p' expresses a physician's opinion about the likelihood of an event as a number between 0 and 1. An event that is certain to occur has a probability of 1; an event that is certain not to occur has a probability of 0. After the pretest probability of disease has been estimated, the second stage of the diagnostic process involves gathering more information, often by performing diagnostic tests. The tests are prescribed to the patients to reduce uncertainty about the diagnosis of the disease. Whenever a test is performed, based on the results of the test the posttest probability is updated. The negative / positive test results support the diagnosis of disease and hence the uncertainty is reduced. The posttest probability of the previous test will be the pretest probability of the next test. The next test is based on the pretest probability. As the posttest probability of the particular trajectory gradually increases that trajectory leads to optimal decision.



Process of Medical Diagnosis

The first stage involves making an *initial judgment* about whether a patient is likely to have a disease. After an interview and physical examination, a physician intuitively develops a belief about the likelihood of disease. This judgment may be based on the previous experience or on knowledge of the medical literature. A physician's belief about the likelihood of disease usually is implicit; he can refine it by making an explicit estimation of the probability of disease. This estimated probability, made before further information is obtained, is the prior probability or pretest probability of the disease. This process of diagnosis involves decision making in three different areas [6].

- With initial symptoms and examinations, the physician may postulate more than one diagnostic hypothesis.
- The physician often narrows down the list and chooses to pursue one hypothesis at a time. The choice is made based on the probability of the presence of the disease and the urgency in treating it. It also depends on the expertise of the physician, and his prior experience in a similar case, etc.
- The physician uses clinical findings to confirm the selected hypothesis. This is usually done by ordering diagnostic tests and / or procedures. Depending on the disease, there could be a wide number of choices of tests / procedures available to confirm the hypothesis. The physician decides on the strategy to choose the hypothesis.
- Using the results of the tests and / or procedures, the physician has to decide either to accept or discard the current hypothesis.

Determining the most efficient use of diagnostic tests is one of the complex issues the medical practitioners face in the process of diagnosis. It is generally accepted that excessive use of tests is a common practice in medical diagnosis. Many tests are performed even though the incremental knowledge gained does not affect the course of diagnosis. Various decision making tools assisting physicians in diagnosis management have been presented by the research scholars. Most of the decision-analytic methods are based on Bayes' theory and decision trees [2].

There are two basic approaches of ordering diagnostic test and/or procedures:

- Parallel approach
- Sequential approach

In parallel testing, all the tests are ordered at once and then from the data obtained, the physician deduces the most likely diagnosis. This method is more costly as the knowledge obtained from the tests may be redundant. It may also put strain on the institution's resources. In a sequential approach, physician orders tests and / or procedures one at a time. The results are obtained and analyzed and its effect on the process of diagnosis documented. There is a possibility of confirmation of the diagnostic hypothesis by the test / procedure prescribed by the physician. If not, the physician may prescribe more tests.

Diagnosis of a disease and its treatment are not separate instead, they are very often dependent and interleaved over time [19]. This is mostly due to uncertainty about the underlying disease, uncertainty associated with the response of a patient to the treatment and varying cost of different diagnostic (investigative) and treatment procedures. Although the correct diagnosis helps to narrow the appropriate treatment choices, it is often the case that the treatment must be pursued without knowing the underlying patient state with certainty. The reason for this is that the diagnostic process is not a one-shot activity and it is usually necessary to collect additional information about the underlying disease, which intern may delay the treatment and make the patients' outcome worse. This process may be even more complex when uncertainty is associated with the reaction of a patient to different treatment choices affecting cost wise with various diagnostic (investigative) actions. This needs to be considered.

Thus, in a course of patient management one needs to carefully evaluate the benefit of possible diagnostic (investigative) and treatment steps and their prescription with regard to the overall global objective, the well being of a patient. To model accurately the complex sequential decision process that combines diagnostic and treatment steps, a framework that is expressive enough to capture all relevant features of the problem is required. The tools typically used to model and analyze decision processes are (stochastic) decision trees [20]. The key drawback of stochastic trees is that they require a large number of parameters to be defined, and, so they are hard to construct and modify. The purpose is too narrow and it is hard to apply them to other tasks, e.g. predictions or explanations. More compact decision models are typically used to alleviate the complexity problem.

In the study, we will employ few terms are used in the context of medical diagnosis which requires definition.

Definition: An *attribute* is a sign or symptom which can provide information for the medical diagnosis. An attribute is binary-valued (it is either present or absent).

Definition: A *test* is the means employed to detect the presence or absence of one or more attributes.

For example, the test for the attribute "family history of a patient" might be a simple question. The test for some other attribute may be a laboratory procedure. Those aspects of diagnosis which are concerned with the selection of a test or sequence of tests will be referred to collectively as the test selection function.

Definition: The *Function of the Clinical Laboratory* is to provide support to the clinician for the purpose of diagnosing disease and monitoring treatment. The data generated by the laboratory, and the interpretation of this data, assist the clinician in the diagnosis.

Definition: A *Medical Diagnostic Problem* is one which is given a set of attributes demanding explanations for such attributes.

Medical diagnosis can be stated as follows: given a set of symptoms and the results of the tests performed on the patient, assess pathological situations identifying those diseases which justify the particular findings. Diagnosis, then consists of two major functions, inference and test selection. A more complete model of diagnosis must provide for the interaction of these two functions. A physician seldom has sufficient information initially to make a satisfactory diagnosis. He can then use the information at hand, however, to form a tentative view of the problem. Using this current view of the problem in conjunction with his medical knowledge and experience, he can select a testing strategy which he expects to yield significant information. Since he often performs these tests sequentially, he has the opportunity to modify his intended testing strategy in the light of new and perhaps unexpected test results. A model of diagnosis should reflect these possibilities.

In considering a particular test, the physician should weigh the expected value of the test results against the expected cost of the test. Because tests can be costly (in terms of patient discomfort, time, money, etc.). Diagnostic tests should be kept to a minimum. On the other hand, the physician seeks to minimize the consequences of possible inappropriate diagnoses. In general, the probability of such

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an error is reduced as more information about the patient is available. Hence the physician may wish to perform a large number of tests to reduce his uncertainty about the condition of the patient. Because these are two contradictory objectives, he must strike a balance between the two. In this view, an appropriate model of diagnosis is 'one which reflects this sequential decision problem confronting the physician.

From the above descriptions, a Diagnostic Problem (DP) of any system can be defined as a combination of four components as follows:

$DP = (D, S, R, S')$ Where

$D = \{ d_1, d_2, ..., d_m \}$

be the set of all disorders / diseases d_i of a system

$S = \{ s_1, s_2, ..., s_n \}$

be the set of all symptoms that may occur when on or more disorders are present

R is a relation between D and S. ie. $R \to D \times S$ and $R \subseteq D \times S$.

Domain $(R) = D = \{ d_i / (d_i, s_i) \in R \}$ for some d_i and

Range $(R) = S = \{ s_k / (d_i, s_k) \in R \}$ for some s_k

$R = \phi$ (null) only if both D and S are empty

$S' \subseteq S$ consisting of the symptoms occurring in the specific case.

In medical diagnosis D represents set of all known diseases and S represents all possible symptoms of a patient. The relation R captures the intuitive notion of causation, where $(d_i, s_i) \in R$ implies that disease d_i can cause s_i. Note that $(d_i, s_i) \in R$ does not imply that s_j always occurs when d_i is present, but only that s_i may occur. For example, a patient suffering from heart attack may have chest pain, numbness in the left arm, sweating, loss of consciousness, or any of several other symptoms, but none of these symptoms are necessarily present. In such situations, logical expression constructed using logical connectives such as conjunction $(\wedge)$, disjunction $(\vee)$, and negation $(\neg)$ can be used to formulate diagnostic rules by connecting different combinations of symptoms.

Consider the following cases:

Case (1): $d_i = \{ s_1 \wedge s_2 \wedge ...... \wedge s_k \}$

where $s_1, s_2, ... s_k$ are the symptoms of the disease d_i and all the symptoms are simultaneously present.

Case (2): $d_i = \{ s_1 \wedge s_2 \wedge (\neg\ s_3) ...... \}$

where $s_1, s_2, s_3, ...$ are some of the symptoms of the disease d_i. In this case symptoms s_1 and s_2 are present and the symptom s_3 is not present.

Case (3): $d_k = \{ s_1 \vee s_2 \}$ where s_1 and s_2 are the symptoms of the disease d_k but one of them is present.

From the above it is clear that

$$sym(d_i) = \{ s_j / (d_i, s_j) \in R \},$$

for every $d_i \in D$ and

$$causes(s_j) = \{ d_i / (d_i, s_j) \in R \}$$

for some d_i, for every $s_j \in S$

These sets represent all possible symptoms caused by d_i and all possible disorders that cause s_i respectively. These concepts are intuitively familiar to the human diagnostician. According to medical science, if $sym(d_i)$ is known for every disorder d_i or if $causes(s_j)$ is known for every symptom s_j, then the causal relation R is completely determined and vice versa. That is

$$sym(D) = \cup sym(d_i) \text{ and}$$

$$causes(S) = \cup causes(s_j)$$

to indicate all possible symptoms of a set of disorders D and all possible causes of any symptom in S respectively. The above is abbreviated as follows.

$$sym'(d_i) = sym(d_i) \cap S' \text{ and}$$

$$sym(D) = sym(D) \cap S'$$

Thus, $sym'(d_i)$ represents that subset of S' which could be caused by d_i.

In the study it is to be noted that the disease does not cause any of the expected symptoms of the disease and is treated as a worst case and hence ignored.

Markov Decision Process for Diagnosis:

A framework more suitable for modeling the medical diagnostic problem is Partially Observable Markov Decision Process (POMDP) [12, 5]. A POMDP model is more compact and thus easier to build and modify than a decision tree [14]. Unfortunately, the modeling power of POMDP is limited with diagnostic problems of small complexity [13]. A challenging goal in the research area is to find alternative structural properties of the domain and suitable approximations that perform well which can be used to obtain good solutions efficiently. This can be best achieved by using Stochastic Finite Automata.

Modeling Medical Diagnosis Using Stochastic Finite Automata

The aim is to develop a model based on Stochastic Finite Automata for finding a strategy for optimal medical diagnosis. The method, based on the patient parameters (both observed and tested), recommends test(s) with the objective of optimizing a measure of performance for the diagnostic process. The performance measure is a combination of cost of the testing, the risk and discomfort associated with the tests, the time taken to reach diagnosis and the diagnostic ability of the tests [15]. The methodology is developed combining tools from mathematical functions and relations, utility theory and automata theory. Functions and relations are used in extracting diagnostic information in the form of rules from the medical databases. Utility theory is used to understand the three non-homogenous measures (cost of testing, risk and discomfort and diagnostic ability) into one value based measure of performance. The Stochastic Finite Automata framework facilitates obtaining efficient testing strategies.

This is very important in the field of medicine, where one must consider not only the end-outcome of a treatment, but also the means to achieve it. For example, different procedures carry different costs in terms of discomfort to the patient and potentially economic cost. Similarly, during the treatment, a patient can end up in different intermediate states with various possible complications and with different levels of pain, suffering and discomfort. Then, even if the end-state is the same, the paths are different. Diagnostic tests extend the information base beyond what is usually obtained through a physical examination and medical history. They allow the physician to obtain a deeper insight of the patient's medical condition. New and advanced diagnostic tests and tools are constantly being introduced to facilitate better understanding and treatment of disease or acquired abnormalities.

Stochastic Finite Automata

A finite state automaton is a model of computation consisting of a set of states, an input alphabet, an initial state, a set of transition rules and a set of final states. Transitions are the rules in the following form:

if (current-state) and (condition) **then** (activate-new-state)

The transitions rules may be given by a function or a relation, mapping or relating the current state and the actual input symbol to the next state. A finite state automaton can decide whether an input string is accepted or not. To this end, the finite state automaton performs a computation beginning with the initial state reading the first symbol from the input string. The computation consists of a series of transitions. In each transition, the next input symbol is read from the input string and the current state is changed according to the transition rules to establish a new state. The computation terminates when the automaton has read the last symbol from the input string. The automaton will accept the input string if it terminates in an accepting / final state.

A Finite Automaton can be defined mathematically as follows [7].

Definition: A finite state automaton is a five-tuple $M = (Q, \Sigma, \delta, q_0, F)$

Where,

Q : a non empty finite set of states

Σ : a finite set of input symbols.

$\delta : Q \times \Sigma \rightarrow Q$: a transition function (δ) mapping from $Q \times \Sigma$ into Q. i.e., from a state $q \in Q$ on taking an input symbol $s \in \Sigma$ there exists transition(s) to another state / states

$q_0 \in Q$: start state or initial state.

F : a set of final states or accepting states and $F \subseteq Q$

The language of the automaton is known as Regular Language which is the set of all accepted input strings over the input alphabet.

A finite state automaton is called stochastic if the transition rules are defined by transition probabilities and initial and final states are defined by probability distributions. A stochastic finite state automaton M^* is pair (M, p) can be defined as follows.

$M^* = (Q, A, q_0, \delta, p, F)$ where

Q is a non empty finite set of states

A is a non empty set of actions

$q_0 \in Q$ is the initial state

$F \subseteq Q$ is a set of final states

$\delta \subset Q \times A \times Q$ is a finite set of transitions between states and

p is a function $\delta \rightarrow [0,1]$ such that for all $q \in Q$ and for all $a \in A$, $\sum_{q' \in Q} p(q, a, q') = 1$

The function p can be generalized to $p^* : Q \times A^* \times Q \rightarrow [0,1]$ in the following way:

$$p^*(q, a, q') = \begin{cases} 1 & if \quad q = q' \\ 0 & if \quad q \neq q' \end{cases}$$ and

$$p^*(q, xa, q'') = \sum_{q' \in Q} p^*(q, x, q') \circ p(q', a, q'')$$ for all $x \in A^*$ and $a \in A$ and $q, q', q'' \in Q$

A stochastic finite state automaton M^* induces a function $\wp : A^* \rightarrow [0,1]$ as follows:

for all $x \in A^*$, $\wp(x) = \sum_{q \in F} p^*(q_0, x, q)$.

Therefore, M^* also induces a weighted language in A^*. It has already been established that the Stochastic Regular Language as a Mathematical Model for the Language of Sequential Actions for Decision Making under Uncertainty [4].

PTS can be extended using the outcomes O and when treated with the transition probability of an action as the transition probability of the outcomes of an action it may be called as Modified Probability Transition System (MPTS).

Definition: MPTS is a five tuple (S, A, O, Tr, p) in such a way that the transition probability distribution p is as

$$p : S \times A \times O \times S \rightarrow [0,1] \text{ such that}$$

$$p = \begin{cases} p(s, a, o, s'), & if \ (s, a, o, s') \in Tr \\ 0, & otherwise \end{cases}$$

and $\Sigma p(s, a, o, s') = 1$ for all $s \in S$ and $o \in O$

The transition relation is defined as $Tr \subseteq S \times A \times O \times S \rightarrow [0,1]$,

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$p(s,a,o,s')$ can be written as $s \xrightarrow{a,o,p} s'$. If p is the transition probability function given above, we can define for all $s \in S, a \in A, o \in O, S' \subseteq S; p(s,a,o,s')$

$$= \Sigma\left\{ p \mid s \xrightarrow{a,0,p} s', s' \in S' \right\}$$

This can also be called as Modified Probabilistic Transition System (MPTS).

Using MPTS, Stochastic Finite Automata can be extended by adding two components O and U and it is a pair (M,P) such that $M = (Q, A, \delta^*, q_0, O, U, F)$ where $\delta^* \subset Q \times A \times O \times U \times Q$ is a finite set of transitions between states and P is a function $\delta^* \to [0,1]$ which means that probability can be assigned to the occurrence of the outcomes as $R \to S \times D$, R is a relation between the symptoms and diseases and the clinical tests/ diagnostic procedures depend on R, the test T can be represented as $T = f(R)$. Hence, the extended Stochastic Finite Automaton can be modified by replacing A by T in order to model medical diagnosis problem under uncertainty. Hence, the mathematical model for medical diagnosis problem under uncertainty is a seven tuple defined as follows

$M = (Q, T, \delta^*, q_0, O, U, F)$ where

Q is a finite set of states

T is a non empty set of clinical tests

$q_0 \in Q$ is the initial state

$F \subseteq Q$ is a set of final states

$\delta \subset Q \times T \times Q$ is a finite set of transitions between states and p is a function $\delta \to [0,1]$ such that for all $q \in Q$ and for all $t \in T$

$$\sum_{q' \in Q} p(q,t,q') = 1$$

The function p can be generalized to $p^*: Q \times T^* \times Q \to [0,1]$ in the following way:

$$p^*(q,t,q') = \begin{cases} 1 & if\ q = q' \\ 0, & if\ q \neq q' \end{cases}$$ and

$$p^*(q, xa, q'') = \sum_{q' \in Q} p^*(q,a,q') \circ p(q',a,q'')$$ for all $x \in A^*$ and $a \in A$ and $q, q', q'' \in Q$

A stochastic finite state automaton M^* induces a function $\wp : A^* \to [01,]$ as follows:

for all $x \in A^*$, $\wp(x) = \sum_{q \in F} p^*(q_0, x, q)$

Optimality of Medical Diagnostic Problem

Presently, the use of diagnostic tests is rapidly increasing. The impact of a diagnostic test is felt mostly by the patient. Patients look for safety and efficacy more than the cost.. A safe test does not cause an unacceptable degree of direct harm to the patient. The efficacy of the diagnostic test is measured in terms of its safety, its technical quality, its therapeutic impact and its impact on the health of the patient [20]. Medical Diagnostic Problems are stochastic in nature. As nature can behave stochastically diagnostic action (say clinical test) it can result in more than one outcome. The outcomes are probabilistically distributed. The outcomes carry numerical values representing rewards or utilities for the corresponding path [21]. The objective of a physician is to choose a test that optimizes rewards (or utilities) associated with outcomes. The optimality criterion used is based on expectations; that is, the objective is to maximize expected reward of possible trajectories.

The optimal test at the initial state s_0 (say) of the trajectory is

$$\mu^*(s_0) = \arg\max_{a \in A} E(r|s_0, a)$$

$$= \arg\max_{a \in A} \sum_{o \in O} P(o|s_0, a) r(\{s_0, a, o\}),$$

where T is a set of clinical tests and $t \in T$, r is a reward, o is the outcome (observation), O is a set of possible outcomes (observations) and $o \in O$ and $r(s_0, t., o)$ is a reward associated with the path (s_0, t, o). In this model, the decision-maker can move more than once and outcomes of all the choices can be stochastic.

The optimal strategy for the multi-step sequential clinical tests is more complex and includes a sequence of choices which are conditioned on the outcomes of earlier steps. Let $V^*(x)$ denote the optimal expected reward for a trajectory starting at the decision node x. V^* is referred as the value function. Given V^*, the optimal value and the optimal choice for any decision state y can be expressed in terms of its successor nodes as:

$$V^*(Y) = \max_{a \in A} \sum_{x_i \in Next\ (y,a)} P(x_i / a, y) V^*(x_i)$$

$$\mu^*(Y) = \arg\ \max_{a \in A} \sum_{x_i \in next\ (y,a)} P(x_i / a, y) V^*(x_i)$$ where $Next\ (t, y)$ is a set of all successors of y and clinical test t. The value of a state equals the reward associated with the corresponding path

$$V^*(x) = r\ (path\ to\ x).$$

Although correct diagnosis helps to narrow the appropriate treatment choices, it is often the case that treatment must be pursued without knowing the underlying patient state with certainty. The reason being the diagnostic process is not a one shot activity and it requires additional information about the underlying disease or problem. This may delay the treatment and make the patient's outcome worse. The process is often made more complex by the uncertainty associated with the patient's reaction to different treatment choices and also the high costs associated with various diagnostic alternatives. Thus, in the course of patient management, one needs to carefully evaluate the benefits of possible diagnostic and treatment steps with regard to patient's well being.

Conclusion

In a medical diagnostic task physicians cannot directly observe the internal state (hidden state) of the patient but rather perceives the symptoms (observations). Yet, after observing every symptom he has to diagnose the disease in a way by means of minimizing the clinical tests, cost, pain and discomfort. Wrong diagnosis and treatments can produce side effects and also be fatal because they can change the internal state of the patient in an unexpected manner. Hence, the goal of the physician is to come up with an optimal diagnosis and treatment plan that will cure the patient. The immediate task of the physician is also to decide the nature of the treatment which is an instance of the problem of sequential decision-making under uncertainty. Thus the stochastic model introduced in the study provides all possible decision policies and theoretical overview for choosing optimal decision.

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