STUDY OF BULK SERVICE QUEUEING MODEL WITH REMOVABLE SERVER

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ABSTRACT

In this chapter we discuss a Markovian Queueing system in which service is performed by a single server in batches. The server is removed from the system as soon as the system becomes empty for a duration which is exponentially distributed. Such situations are not uncommon in our daily life. For example –The situation in a Cinema Hall, the person taking ticket one by one and enter the Hall, after the end of the movie person left the hall with random groups. This type of situation can also observe in computer system; we enter the data one by one i.e. single service and gain the out put in batches.

Keywords— Queue length, busy period, single server, joint distribution, Laplace transformation.

Introduction

In this paper we will study the concept of a bulk service queueing system in which service is performed by single server inn batches. Medhi (1984) discuss the idea of the batch size (fixed batch size or variable batch); it depends on the unfilled capacity of the server. Bailey (1954) considered that the service batch may be fixed size-say, K. The server waits until there are K in the queue and starts service as soon as the queue reaches this size. If on completion of a batch service, he finds more than K waiting, the server takes a batch of size k (in order of arrivals or in any other order), While others, in excess of K units, and wait in the queue.

The following assumptions describe the system:-

1 Arrivals occur under Poisson, law with parameterλ.
2 The queue discipline is FCFS.
3 The service time distribution is exponential with parameterμ.
4 The various stochastic processes in the system are statistically independent.
5 Services occur in batches of variable size. Service times are exponentially distributed with parameterμ.
6 The server will be removed from its service as soon as it becomes empty, for exponential distribution, with parameterθ.

Notation

$P_{i,j,R}(t)$ - Prob. that there are exactly i arrivals and

time t departures by and the server is in

removed state.

$P_{i,j,B}(t)$ - Prob. that there are exactly i arrivals and

j t and departure by time server is busy.

P_{ij} - Prob. that there are exactly i and j departures by time 't'; i >, j >, 0.

Initial conditions

$P_{0,0,R}(0) = 1$

$P_{0,0,B}(0) = 0$

The difference - differential equations governing the system are:

$$p'_{iiR}(t)=-\lambda p_{iiR}(t)+\sum_{l=1}^{i}\mu b_l p_{i,i-l,B}(t)(1-\delta_{i0})$$

…(1)

$$p'_{iiR}(t)=-\lambda p_{iiR}(t)+\sum_{l=1}^{l=1}\mu b_l p_{i,i-l,B}(t) \qquad i^3 b \;\; i>b$$

…(2)

$$p'_{ijR}(t)=-(\lambda+\theta)p_{i,j,R}(t)+\lambda p_{i-1,j,R}(t)$$

…(3)

$$p'_{i,j,B}(t)=-(\lambda+\mu)p_{ijB}(t)+\sum_{l=1}^{j}\mu b_l p_{i,j-l,B}(t)+\lambda p_{i-1,j,B}(t)(1-\delta_{i-1,j})+\theta p_{i,j,R}(t)$$

$0 < i < j,\; i > j$

…(4)

Take L.T. of eq. (1) to eq. (4)

From eq. (1)

$$S\bar{p}_{iiR}(s)-p_{iiR}(0)=-\lambda\bar{p}_{iiR}(s)+\mu\sum_{l=1}^{i}b_l\bar{p}_{i,i-l,B}(s)(1-\delta_{i0})$$

$$(s+\lambda)\bar{p}_{iiR}(s)=p_{iiR}(0)+\mu\sum_{l=1}^{i}b_l\bar{p}_{i,i-l,B}(s)(1-\delta_{i0})$$

$$\bar{P}_{00R}(s)=\frac{1}{(s+\lambda)}$$

…(5)

From Eq. (2)

$$S\bar{p}_{iiR}(s)-p_{iiR}(0)=-(\lambda+\theta)\bar{p}_{i,jR}(s)+\lambda\bar{p}_{i,i-l,B}(s)$$

$i = 1,\; j = 0$

$$(s+\lambda+\theta)\bar{p}_{10R}(s)=\lambda\bar{p}_{0,0,R}(s)$$

$$\bar{p}_{10R}(s)=\frac{\lambda}{s+\lambda+\theta}\bar{p}_{0,0,R}(s)$$

$i = 2,\; j = 0$

$$S\bar{p}_{2,0,R}(s)-p_{2,0,R}(0)=-(\lambda+\theta)\bar{p}_{2,0,R}(s)+\lambda\bar{p}_{1,0R}(s)$$

$$(s+\lambda+\theta)\bar{p}_{20R}(s)=\lambda\bar{p}_{1,0,R}(s)$$

$$\bar{p}_{20R}(s)=\frac{\lambda}{s+\lambda+\theta}.\bar{p}_{1,0,R}(s)$$

$$\bar{p}_{2,0,R}(s)=\frac{\lambda}{(s+\lambda+\theta)}.\frac{1}{(s+\lambda)}.\frac{\lambda}{(s+\lambda+\theta)}$$

$$S\bar{p}_{3,0,R}(s)-p_{3,0R}(0)=-(\lambda+\theta)\bar{p}_{3,0,R}(s)+\lambda\bar{p}_{2,0,R}(s)$$

$$(s+\lambda+\theta)\bar{p}_{3,0,R}(s)=\lambda\bar{p}_{2,0,R}(s)$$

$$\bar{p}_{3,0,R}(s)=\frac{\lambda}{(s+\lambda+\theta)}\bar{p}_{2,0,R}(s)$$

$$\bar{p}_{3,0,R}(s)=\frac{\lambda}{(s+\lambda+\theta)}.\frac{\lambda^2}{(s+\lambda)(s+\lambda+\theta)^2}$$

$$\bar{p}_{3,0,R}(s)=\frac{\lambda^3}{(s+\lambda)(s+\lambda+\theta)^3}$$

$$\bar{p}_{i,0,R}(s)=\frac{\lambda^i}{(s+\lambda)(s+\lambda+\theta)^i}$$

Or

$$\bar{p}_{i,0,R}(s)=\lambda^i\left\{\beta_{1,i}^{\lambda(\lambda+\theta)}(s)\right\} i>0$$

…(.6)

From (1)

$$S\bar{p}_{iiR}(s)=-\lambda\bar{p}_{iiR}(s)+\sum_{l=1}^{i}\mu b_l p_{i,i-l,B}(s)$$

$$(s+\lambda)\bar{p}_{iiR}(s)=\sum_{l=1}^{i}\mu b_l p_{i,i-l,B}(s)$$

$$\bar{p}_{iiR}(s)=\left(\frac{\mu}{s+\lambda}\right)\sum_{l=1}^{i}b_l\bar{p}_{i,i-l,B}(s) \quad ...(7)$$

$$b>i\geq 1$$

From eq. (2)

$$S\bar{p}_{iiR}(s)=-\lambda\bar{p}_{iiR}(s)+\sum_{l=1}^{b}\mu b_l\bar{p}_{i,i-l,B}(s)$$

$$(s+\lambda)\bar{p}_{iiR}(s)=\sum_{l=1}^{b}\mu b_l\bar{p}_{i,i-l,B}(s)$$

$$\bar{p}_{iiR}(s)=\frac{\mu}{s+\lambda}\sum_{l=1}^{b}b_l\bar{p}_{i,i-l,B} \quad i\geq b \quad ...(8)$$

$$\bar{p}_{i0R}(s)=\lambda^i.\frac{1}{(s+\lambda)(s+\lambda+\theta)^i}$$

From Eq. (1)

$$S\bar{p}_{i0R}(s)-p_{iiR}(0)=-\lambda\bar{p}_{iiR}(s)+\mu\sum_{l=1}^{i}b_l\bar{p}_{ii-l,B}(s)(1-\delta_{i0})$$

$$(s+\lambda)\bar{p}_{iiR}(s)=\mu\sum_{l=1}^{i}b_l\bar{p}_{i,i-l,B}(1-\delta_{i0})$$

$$\bar{p}_{i,1,R}(s)=\sum_{l=1}^{j}\lambda^{i-1}\mu b_l.\frac{1}{(s+\lambda).(s+\lambda+\theta)^{i-1}}\quad\bar{p}_{j,1-l,B}(s)$$

$$\bar{p}_{i,2,R}(s)=\sum_{l=1}^{j}\lambda^{i-2}\mu b_l.\frac{1}{(s+\lambda)(s+\lambda+\theta)^{i-2}}\bar{p}_{i,2-l,B}(s)$$

$$\bar{p}_{i,j,R}(s)=\sum_{l=1}^{j}\lambda^{i-j}\mu b_l.\frac{1}{(s+\lambda)(s+\lambda+\theta)^{i-j}}\bar{p}_{j,i-l,B}(s)$$

$$i>j>0 \qquad j<b$$

...(9)

$$\bar{p}_{i1R}(s)=\sum_{l=1}^{j}\lambda^{i-1}\mu b_l\frac{1}{(s+\lambda)(s+\lambda+\theta)^{i-j}}\bar{p}_{j,j-l,B}(s)$$

$$\bar{p}_{i,2,R}(s)=\sum_{l=1}^{b}\lambda^{i-2}\mu b_l\frac{1}{(s+\lambda)(s+\lambda+\theta)^{i-2}}\bar{p}_{j,2-l,B}(s)$$

$$\bar{p}_{i,3,R}(s)=\sum_{l=1}^{b}\lambda^{i-3}\mu b_l.\frac{1}{(s+\lambda)(s+\lambda+\theta)^{i-3}}\bar{p}_{j,3-l,B}(s)$$

$$\bar{p}_{i,j,R}(s)=\sum_{l=1}^{b}\lambda^{i-j}\mu b_l\frac{1}{s+\lambda(s+\lambda+\theta)^{i-j}}\bar{p}_{j,j-l,B}(s)$$

$$\bar{p}_{i,j,R}(s)=\sum_{l=1}^{b}\lambda^{i-j}\mu b_l.\beta_{1,i-j}^{\lambda(\lambda+\theta)}\bar{p}_{j,j-l,B}(s)$$

...(10)

$$\bar{p}_{10B}(s)=\lambda\theta\frac{1}{(s+\lambda)(s+\lambda+\mu)(s+\lambda+\theta)}$$

$$\bar{p}_{2,0,B}(s)=\frac{\lambda^2\theta}{(s+\lambda)(s+\lambda+\mu)(s+\lambda+\theta)^2}$$

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$$\bar{p}_{i,0,B}(s)=\lambda^{i}\theta\sum_{K=0}^{i-1}\frac{1}{(s+\lambda)(s+\lambda+\mu)^{1-K}(s+\lambda+\theta)^{K+1}}$$

$$\bar{p}_{i0B}(s)=\lambda^{i}\theta\sum_{K=0}^{i-1}\beta_{1-K,K+1,1}^{(\lambda+\mu)(\lambda+\theta)}(s)$$

$$\bar{p}_{ijB}(s)=\sum_{l=1}^{j}\sum_{K=0}^{i}\left(\lambda^{i-K}\mu\right)\left\{\frac{b_l}{(s+\lambda+\mu)^{i+1-K}}\right\}\left\{\theta(b_{l=1})\sum_{l=1}^{i-K}\bar{\beta}_{i,i+1-K,l}^{(\lambda+\mu)(\lambda+\theta)}(s)\right\}\bar{p}_{K,i-l,B}$$

i > j > 0

Taking Laplace inverse transformation of equation (5),(6), (7),(8), (9)

$$p_{00R}(t)=e^{-\lambda t}$$
…(11)

$$p_{i0R}(t)=\lambda^{i}\beta_{1.j}^{\lambda(\lambda+\theta)}(t)$$
…(12)

$$p_{iiR}(t)=\frac{\mu}{s+\lambda}\sum_{l=1}^{b}b_l p_{i,i-l,B}(t)$$
…(13)

$$p_{i,jR}(t)=\mu.\lambda^{i-j}\sum_{l=1}^{j}b_l.\beta_{1,i-j}^{\lambda(\lambda+\theta)}p_{j,j-l,B}(t)$$ …(14)

$$p_{iiR}(t)=\frac{\mu}{s+\lambda}\sum_{l=1}^{i}b_l.p_{i,i-l,B}(t)$$
…(15)

$$p_{ijR}(t)=\sum_{l=1}^{j}\lambda^{i-j}.\mu b_l.\beta_{i,i-j}^{\lambda(\lambda+Q)}(t).p_{j,j-l,B}(t)$$ …(16)

$$\bar{p}_{i0B}(t)=\left(\lambda^{i}\theta\right)\sum_{K=0}^{i-1}\beta_{1,i-K,K+1}^{(\lambda+\mu)(\lambda+\theta)}(t)$$
…(17)

$$\bar{p}_{i,i,B}(t)=\sum_{K=0}^{i}\sum_{l=1}^{j}\left(\lambda^{1-K}.\mu\right)b_l\left(\frac{e^{-(\lambda+\mu)t}t^{i+K}}{\underline{|i+K}}\right)\left\{\theta b_l\sum_{l=1}^{i-K}\beta_{1,i+1-K,l}^{(\lambda+\mu)(\lambda+\theta)}(t)\right\}p_{K,j-l,B}(t)$$

From the Laplace equation

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left\{\bar{p}_{ijR}(s)+\bar{p}_{ijB}(s)(1-\delta_{ij})\right\}=\frac{1}{S}$$

From the Laplace inverse transformation

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left\{p_{ijR}(t)+p_{ijB}(t)(1-\delta_{ij})\right\}=1$$

Hence the verification

1) Exactly i units arrive by time t is

$$\bar{p}_{i0}(s)=\sum_{j=0}^{i}\left\{\bar{p}_{ijR}(s)+\bar{p}_{ijB}(s)(1-\delta_{ij})\right\}$$
…(18)

$$\bar{p}_{i0}(s)=\frac{\lambda^{i}}{(s+\lambda)^{i+1}};i>0$$

Laplace inverse of Eq. 18 is

$$p_{i0}(t)=\left\{\frac{(\lambda t)^{i}}{\underline{|i}}.e^{-\lambda t}\right\}$$

The arrivals follow a Poisson distribution the total number of arrivals is not effected by vacation time of the server that is ‘θ’.

2) The mean number of arrivals are

$$\sum_{i=0}^{\infty}i\bar{p}_{i0}(s)=\frac{\lambda}{s^{2}}$$
…(19)

Laplace inverse of Eq. 19

$$\sum_{i=0}^{\infty}ip_{i0}(t)=\lambda(t)$$

This shows the mean number of arrival in time (t).

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