

Pulsed Laser Heating of a Two-layer System in Relation to the Pulse Parameters

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Abstract:

Pulsed laser heating of a two layer system is studied, taking into consideration the temperature dependence of the optical absorption coefficient of the front surface of the irradiated system. Laplace Integral Transform Technique is applied to get the required solutions.

The effect of the laser pulse parameters on the thermal response of the system is revealed. Computations concerning Silicon-Glass system are made as an illustrative example.

The general shape of the laser pulse affects the thermal response of the irradiated target. The parameters of predominant effect on the thermal response are: the maximum value of the laser power density and the time required to reach this value.

Moreover, the effect of the temperature dependence of the irradiated surface is revealed.

Keywords: Pulsed laser heating, laser pulse parameters, two layer system, Laplace integral transform, temperature dependent absorptance.

1. Introduction:

Laser-solid interaction represents a serious problem that has been the subject of research of many authors [1-36]. The problem has important applications. For example, high power density laser source are used in the field of material processing, such as spot welding, scribing, drilling of holes, cutting, shock hardening and glazing of materials [1-8].

In the semiconductor industry, laser is used to form p-n junctions.

Pulsed lasers have been also successfully applied for radiation damage associated with ion implantation in Si during device fabrication [12]. The laser pulse shape is still not universal. Different trials are given to predict the laser pulse [12, 27, 29, 33, 34, and 35].

In the present trial, the laser pulse predicted by the authors elsewhere [34] is accepted in studying the heating problem of a two layer system formed of a thin film deposited on a thick semi-infinite substrate. In addition, the temperature dependence of the absorption coefficient of the front layer surface of the irradiated target is considered.

This factor is important especially in high-power reflectors made of both metallic and multilayer dielectric coated metals [9, 10].

Neglecting such factor gives overestimated values for the values of the critical time required to initiate damage in the laser heated target.

Laplace integral transform technique is considered to get the required solutions for the considered problem. Computations concerning Silicon-Glass system are made as an illustrative example.

2.Theory:

In setting up the problem it is assumed that the laser pulse $q_0(t)$ W/m² is incident on the front surface of a two layer system, where it is partly absorbed $Aq_0(t)$ and partly reflected.

The optical absorptance coefficient (A) is assumed to depend linearly on the temperature of the front irradiated surface According to the relation [9] :

$$A_f = A_0 + A_1 T(o, t)(i)$$

The thickness of the thin film is "d", m the two layers are in perfect thermal contact.

Two coincident coordinate axes x – and z (x-d), in the direction of the incident laser radiation are considered [25]. The coordinate $x = d$ is the interface between the two layers.

Thermal losses arising from any mode other than convection laser are neglected.

2.1 Derivation of the basic equations:

Parabolic heat conduction equation is considered. It is written as follows:

For the front thin film:

$$\frac{\partial \theta_f(x, t)}{\partial t} = \alpha_f \frac{\partial^2 \theta_f(x, t)}{\partial x^2}, \quad t_m > t > 0, \quad 0 \leq x \leq d \quad (1)$$

$$\frac{\partial \theta_p(z, t)}{\partial t} = \alpha_p \frac{\partial^2 \theta_p(z, t)}{\partial z^2}, \quad t > 0, \quad 0 \leq z \leq \infty \quad (2)$$

Where, $\theta (= T - T_o)$ is the excess temperature relative to the ambient temperature " T_o ", and $\alpha = \lambda/\rho c_p$, is the thermal diffusivity in terms of the thermal conductivity λ (W/mK) and the thermal capacity per unit volume (ρc_p), and ρ (kg/m³) is the material density. C_p (J/kg.K) is the specific heat.

The system of equation (1) and (2) is subjected to the following initial and boundary conditions:

$$\text{ii) } \theta_f(x, 0) = 0$$

$$\text{iii) } \theta_p(z, 0) = 0$$

$$\text{iv) } \theta_p(\infty, 0) = 0$$

$$\text{v) } \theta_p(\infty, t) = 0$$

$$\text{vi) } \text{The condition at the front surface } x = 0$$

$$-\lambda_f \left. \frac{\partial \theta_f(x, t)}{\partial x} \right|_{x=0} = A_f(T) q_o(t) - h_o \theta_f(o, t) \quad (3)$$

Where " h_o " is the heat transfer coefficient at the front surface of the system.

The source laser function is suggested by the author elsewhere [34] in the form:

$$q(t) = q_{\max} \left(\frac{t_d - t}{t_d - t_o} \right)^m \left(\frac{t}{t_o} \right), \quad (4)$$

$$\text{where } m = (t_d - t_o) / t_o \quad (5)$$

At the interface $x = d, z=0$

$$\theta_f(d, t) = \theta_p(o, t) \quad (6)$$

$$\lambda_f \left. \frac{\partial \theta_f(x, t)}{\partial x} \right|_{x=d} = -\lambda_p \left. \frac{\partial \theta_p(z, t)}{\partial z} \right|_{z=0} \quad (7)$$

Together with the integrated heat balance equation in the form:

$$\int_0^t A_f(T) q_o(t) dt = \int_0^d \rho_f C_{pf} \theta_f(x, t) dx + \int_0^\infty \rho_p C_{pp} \theta_p(z, t) dz + \int_0^t h_o \theta_f(o, t) dt \quad (8)$$

Taking Laplace transform with respect to time of equations (1) and (2) considering the condition (ii) and (iii), one gets:

$$S \bar{\theta}_f(x, s) - \alpha_f \frac{\partial^2 \bar{\theta}_f(x, s)}{\partial x^2} = 0 \quad (9)$$

$$S \bar{\theta}_p(z, s) - \alpha_p \frac{\partial^2 \bar{\theta}_p(z, s)}{\partial z^2} = 0 \quad (10)$$

Where $\bar{\theta}$ is the Laplace transform of θ . The solutions of eq. (9) and (10) can be written respectively in the form:

$$\bar{\theta}_f(x, s) = C_1 \exp\left(\sqrt{\frac{s}{\alpha_f}} x\right) + C_2 \exp\left(-\sqrt{\frac{s}{\alpha_f}} x\right) \quad (11)$$

and

$$\bar{\theta}_p(z, s) = C_3 \exp\left(-\sqrt{\frac{s}{\alpha_p}} z\right) + C_4 \exp\left(+\sqrt{\frac{s}{\alpha_p}} z\right) \quad (12)$$

In order to have a finite solution, condition (IV) implies that $C_4 = 0$, and thus the solution in the substrate will attain the form :

$$\bar{\theta}_p = C_3 \exp\left(-\sqrt{\frac{s}{\alpha_p}} z\right) \quad (13)$$

Laplace transform of the conditions gives :

$$\lambda_f \frac{\partial \bar{\theta}_f(x, s)}{\partial x} \Big|_{x=0} = R(s) - h_o \bar{\theta}_f(o, s) \quad (14)$$

Where, $R(s)$ is the Laplace transform as follows :

$$R(s) = L \left\{ (A_1 + A_2 \theta_f(o, t)) q(t) \right\} \quad (15)$$

The heat balance equation transforms to:

$$\frac{R(s)}{s} = \int_0^d \rho_f C_{p_f} \bar{\theta}_f(x, s) dx + \int_0^\infty \rho_p C_{p_p} \bar{\theta}_p(z, s) dz + \frac{h_o \bar{\theta}_f(o, s)}{s} \quad (16)$$

Moreover, the other two conditions at the interface will be in the form:

$$\bar{\theta}_f(d, s) = \bar{\theta}_p(o, s) \quad (17)$$

and

$$-\lambda_f \frac{\partial \bar{\theta}_f(x, s)}{\partial x} \Big|_{x=d} = -\lambda_p \frac{\partial \bar{\theta}_p(z, s)}{\partial z} \Big|_{z=0} \quad (18)$$

Substituting the solutions (11) and (13) into the system of equations (14), (15), (16) (17) and (18) one gets :

$$\left(h_o - \lambda_f \sqrt{\frac{s}{\alpha_f}} - A_{f_2} q_o \right) C_1 + \left(h_o + \lambda_f \sqrt{\frac{s}{\alpha_f}} - A_{f_2} q_o \right) C_2 = R(s) \quad (19)$$

$$\exp\left(\sqrt{\frac{s}{\alpha_f}} d\right) C_1 + \exp\left(-\sqrt{\frac{s}{\alpha_f}} d\right) C_2 - C_3 = 0 \quad (20)$$

$$-\lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(\sqrt{\frac{s}{\alpha_f}} d\right) C_1 + \lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(-\sqrt{\frac{s}{\alpha_f}} d\right) C_2 - \lambda_p \sqrt{\frac{s}{\alpha_p}} C_3 = 0 \quad (21)$$

$$\begin{aligned} & \left[\rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s}} \exp\left(\sqrt{\frac{s}{\alpha_f}} d\right) - \rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s} + \frac{h_o}{s}} \right] C_1 + \\ & \left[-\rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s}} \exp\left(-\sqrt{\frac{s}{\alpha_f}} d\right) + \rho_f C_{p_f} \sqrt{\frac{\alpha_f}{s} + \frac{h_o}{s}} \right] C_2 + \rho_p C_p \sqrt{\frac{\alpha_p}{s}} C_3 = \frac{R(s)}{s} \end{aligned} \quad (22)$$

The system of equation (19 – 22) makes it possible to determine the unknowns C_1 , C_2 and C_3 . They are obtained in the form:

$$\begin{aligned} C_1 = R(s) & \left[\lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(-\sqrt{\frac{s}{\alpha_f}} d\right) - \lambda_p \sqrt{\frac{s}{\alpha_p}} \exp\left(-\sqrt{\frac{s}{\alpha_f}} d\right) \right] \div \\ & \left\{ \left(\lambda_p \sqrt{\frac{s}{\alpha_p}} h_o + \lambda_f^2 \frac{s}{\alpha_f} \right) \sinh\left(\sqrt{\frac{s}{\alpha_f}} d\right) + \left(\lambda_p \lambda_f \sqrt{\frac{s}{\alpha_f}} \sqrt{\frac{s}{\alpha_p}} + h_o \lambda_f \sqrt{\frac{s}{\alpha_f}} \right) \cosh\left(\sqrt{\frac{s}{\alpha_f}} d\right) \right\} \end{aligned} \quad (23)$$

$$\begin{aligned} C_2 = R(s) & \left[\lambda_f \sqrt{\frac{s}{\alpha_f}} \exp\left(-\sqrt{\frac{s}{\alpha_f}} d\right) - \lambda_p \sqrt{\frac{s}{\alpha_p}} \exp\left(-\sqrt{\frac{s}{\alpha_f}} d\right) \right] \div \\ & \left\{ \left(\lambda_p \sqrt{\frac{s}{\alpha_p}} h_o + \lambda_f^2 \frac{s}{\alpha_f} \right) \sinh\left(\sqrt{\frac{s}{\alpha_f}} d\right) + \left(\lambda_p \lambda_f \sqrt{\frac{s}{\alpha_f}} \sqrt{\frac{s}{\alpha_p}} + h_o \lambda_f \sqrt{\frac{s}{\alpha_f}} \right) \cosh\left(\sqrt{\frac{s}{\alpha_f}} d\right) \right\} \end{aligned} \quad (24)$$

$$C_3 = R(s) \left[2\lambda_f \sqrt{\frac{s}{\alpha_f}} \right] \div \left\{ \left(\lambda_\rho \sqrt{\frac{s}{\alpha_\rho}} h_o + \lambda_f^2 \frac{s}{\alpha_f} \right) \sinh \left(\sqrt{\frac{s}{\alpha_f}} d \right) + \right. \\ \left. \left(\lambda_\rho \lambda_f \sqrt{\frac{s}{\alpha_f}} \sqrt{\frac{s}{\alpha_\rho}} + h_o \lambda_f \sqrt{\frac{s}{\alpha_f}} \right) \cosh \left(\sqrt{\frac{s}{\alpha_f}} d \right) \right\} \quad (25)$$

Substituting equations (23-25) into equations (11) and (13) and rearranging the obtained expressions one can obtain the solutions in the form:

$$\bar{\theta}_f(x, s) = R(s) \left\{ \left[\exp \left(-\sqrt{\frac{s}{\alpha_f}} x \right) + \exp \left(-\sqrt{\frac{s}{\alpha_f}} (2d - x) \right) \right] + \frac{\lambda_\rho}{\lambda_f} \sqrt{\frac{\alpha_f}{\alpha_\rho}} \right. \\ \left[\exp \left(-\sqrt{\frac{s}{\alpha_f}} x \right) - \exp \left(-\sqrt{\frac{s}{\alpha_f}} (2d - x) \right) \right] \right\} \div \left\{ \left(\frac{\lambda_\rho h_o}{\lambda_f} \sqrt{\frac{\alpha_f}{\alpha_\rho}} + \lambda_\rho \sqrt{\frac{s}{\alpha_\rho}} + \lambda_f \sqrt{\frac{s}{\alpha_f}} + h_o \right) \right. \\ \left. \left[1 - \gamma \exp \left(-2\sqrt{\frac{s}{\alpha_f}} d \right) \right] \right\} \quad (26)$$

Where,

$$\gamma = \frac{\left(\frac{\lambda_\rho h_o}{\lambda_f} \sqrt{\frac{\alpha_f}{\alpha_\rho}} + \lambda_f \sqrt{\frac{s}{\alpha_f}} \right) - \left(\lambda_\rho \sqrt{\frac{s}{\alpha_\rho}} + h_o \right)}{\left(\frac{\lambda_\rho h_o}{\lambda_f} \sqrt{\frac{\alpha_f}{\alpha_\rho}} + \lambda_f \sqrt{\frac{s}{\alpha_f}} \right) + \left(\lambda_\rho \sqrt{\frac{s}{\alpha_\rho}} + h_o \right)} \quad (27)$$

and

$$\bar{\theta}_s(z, s) = R(s) \left\{ 4 \exp \left(-\left(\frac{z}{\sqrt{\alpha_\rho}} + \frac{d}{\sqrt{\alpha_f}} \right) \sqrt{s} \right) \right\} \div \left\{ \left(\frac{\lambda_\rho h_o}{\lambda_f} \sqrt{\frac{\alpha_f}{\alpha_\rho}} + \lambda_\rho \sqrt{\frac{s}{\alpha_\rho}} + \lambda_f \sqrt{\frac{s}{\alpha_f}} + h_o \right) \right. \\ \left. \left[1 - \gamma \exp \left(-2\sqrt{\frac{s}{\alpha_f}} d \right) \right] \right\} \quad (28)$$

Equation (27) indicates that $0 \leq \gamma \leq 1$

Moreover, one can make use of the relation:

$$\frac{1}{1-a} = \sum_{n=0}^{\infty} a^n, \quad |a| < 1 \quad [38] \quad (29)$$

$$\text{Thus } \frac{1}{1 - \gamma \exp\left(-2\sqrt{\frac{s}{\alpha}}d\right)} = \sum_{n=0}^{\infty} \gamma^n \exp\left(-2n\sqrt{\frac{s}{\alpha}}d\right) \quad (30)$$

This makes it possible to write the equations for $\bar{\theta}_f(x, s)$ and $\bar{\theta}_p(z, s)$ in the following forms:

$$\bar{\theta}_f(x, s) = \frac{R(s)\sqrt{\alpha_f\alpha_\rho}}{\lambda_f\sqrt{\alpha_\rho} + \lambda_\rho\sqrt{\alpha_f}} \left\{ \left[\exp\left(-\sqrt{\frac{s}{\alpha_f}}x\right) + \exp\left(-\sqrt{\frac{s}{\alpha_f}}(2d-x)\right) \right] + \frac{\lambda_\rho}{\lambda_f} \sqrt{\frac{\alpha_f}{\alpha_\rho}} \left[\exp\left(-\sqrt{\frac{s}{\alpha_f}}x\right) - \exp\left(-\sqrt{\frac{s}{\alpha_f}}(2d-x)\right) \right] \right\} \div \left\{ \left(\sqrt{s} + \frac{h_o\sqrt{\alpha_f}}{\lambda_f} \right) \sum_{n=0}^{\infty} \gamma^n \exp\left(-2n\sqrt{\frac{s}{\alpha_f}}d\right) \right\} \quad (31)$$

$$\bar{\theta}_p(z, s) = \frac{R(s)\sqrt{\alpha_f\alpha_\rho}}{\lambda_f\sqrt{\alpha_\rho} + \lambda_\rho\sqrt{\alpha_f}} \left\{ 2 \exp\left(-\left(\sqrt{\frac{z}{\alpha_\rho}} + \frac{d}{\sqrt{\alpha_f}}\right)\sqrt{s}\right) \right\} \div \left\{ \left(\sqrt{s} + \frac{h_o\sqrt{\alpha_f}}{\lambda_f} \right) \sum_{n=0}^{\infty} \gamma^n \exp\left(-2n\sqrt{\frac{s}{\alpha_f}}d\right) \right\} \quad (32)$$

To find the inverse Laplace transform for the expressions equations (35) and (36), one has to apply the convolution theorem (38, 39, 40) written in the form:

$$\int_0^t \{f_1(s) f_2(s)\} = \int_0^t \{f_1(u) f_2(t-u) du\} \quad (33)$$

Moreover, using the standard tables for finding the inverse Laplace transformation [38, 39, 40 & 41]. One finally gets for $\gamma = 1$ the required solutions in the form:

* For the front thin film :

$$\begin{aligned}
\theta_f(x, t) = & \frac{\sqrt{\alpha_f \alpha_p}}{\lambda_f \sqrt{\alpha_p} + \lambda_p \sqrt{\alpha_f}} \left[\int_0^t \left\{ (A_{f1} + A_{f2} \theta_f(o, u)) q(u) \sum_{n=0}^{\infty} \left(\frac{\exp\left(\frac{(2nd+x)^2}{4\alpha_f t}\right)}{\sqrt{\pi(t-u)}} - \frac{h_o \sqrt{\alpha_f}}{\lambda_f} \right. \right. \right. \\
& \exp\left(\frac{h_o(2nd+x)}{\lambda_f}\right) \exp\left(\frac{h_o^2 \alpha_f}{\lambda_f^2} (t-u)\right) \operatorname{erfc}\left(\frac{h_o \sqrt{\alpha_f(t-u)}}{\lambda_f} + \frac{2nd+x}{\sqrt{4\alpha_f(t-u)}}\right) + \\
& \exp\left(-\frac{((2n+2)d-x)^2}{4\alpha_f(t-u)}\right) - \left(\frac{h_o \sqrt{\alpha_f}}{\lambda_f}\right) \exp\left(\frac{h_o(2nd+x)}{\lambda_f}\right) \exp\left(\frac{h_o^2 \alpha_f}{\lambda_f^2} (t-u)\right) \\
& \left. \left. \left. \operatorname{erfc}\left(\frac{h_o \sqrt{\alpha_f(t-u)}}{\lambda_f} + \frac{(2n+2)d-x}{\sqrt{4\alpha_f(t-u)}}\right) \right) \right\} + \frac{\lambda_p}{\lambda_f} \sqrt{\frac{\alpha_f}{\alpha_p}} \right. \\
& \sum_{n=0}^{\infty} \left(\frac{\exp\left(\frac{(2nd+x)^2}{4\alpha_f(t-u)}\right)}{\sqrt{\pi(t-u)}} - \frac{h_o \sqrt{\alpha_f}}{\lambda_f} \exp\left(\frac{h_o(2nd+x)}{\lambda_f}\right) \exp\left(\frac{h_o^2 \alpha_f}{\lambda_f^2} t\right) \right. \\
& \left. \left. \left. \operatorname{erfc}\left(\frac{h_o \sqrt{\alpha_f(t-u)}}{\lambda_f} + \frac{2nd+x}{\sqrt{4\alpha_f(t-u)}}\right) \right) \frac{\exp\left(\frac{((2n+2)d-x)^2}{4\alpha_f(t-u)}\right)}{\sqrt{\pi(t-u)}} + \frac{h_o \sqrt{\alpha_f}}{\lambda_f} \right. \right. \\
& \left. \left. \left. \exp\left(\frac{h_o((2n+2)d-x)}{\lambda_f}\right) \exp\left(\frac{h_o^2 \alpha_f}{\lambda_f^2} (t-u)\right) \right) \right. \right. \\
& \left. \left. \left. \operatorname{erfc}\left(\frac{h_o \sqrt{\alpha_f(t-u)}}{\lambda_f} + \frac{(2n+2)d-x}{\sqrt{4\alpha_f(t-u)}}\right) \right) \right\} du \right]
\end{aligned}
\tag{34}$$

For the substrate :

$$\theta_{\rho}(z,t) = \frac{4\sqrt{\alpha_f\alpha_{\rho}}}{\lambda_f\sqrt{\alpha_{\rho}} + \lambda_{\rho}\sqrt{\alpha_f}} \left[\int_0^t \left(A_{f_1} + A_{f_2} \theta_2(0,u) \right) q(u) \sum_{n=0}^{\infty} \exp \left[\frac{1}{\sqrt{\pi(t-u)}} \left(\frac{\left(\frac{2nd+d}{\alpha_f} + \frac{z}{\alpha_{\rho}} \right)^2}{4(t-u)} \right) \right. \right. \right. \\ \left. \left. \left. - \frac{h_o\sqrt{\alpha_f}}{\lambda_f} \exp \left(\frac{h_o \left(\frac{2nd+d}{\alpha_f} + \frac{z}{\alpha_{\rho}} \right)^2}{4(t-u)\lambda_f} \right) - \exp \left(\frac{h_o^2\alpha_f}{\lambda_f^2} (t-u) \right) \operatorname{erfc} \left(\frac{h_o\sqrt{\alpha_f}t}{\lambda_f} + \right. \right. \right. \right. \\ \left. \left. \left. \frac{2nd+d}{\sqrt{4\alpha_f(t-u)}} + \frac{z}{\sqrt{4\alpha_{\rho}(t-u)}} \right) \right] du \right] \quad (35)$$

Moreover, to obtain the temperature $\theta_f(o,t)$ of the front surface, one has to rewrite equation (34) in the form :

$$\theta_f(o,y) = \int_0^t A_{f_1} q(u) [F(t-u)]_{x=0} du + \int_0^t A_{f_2} \theta_f(o,u) q(u) [F(t-u)]_{x=0} du \quad (36)$$

This is an integral equation of Hammerstein type [44]. Its solution is obtained in the form:

$$\theta_f(o,t) = A_{f_1} \int_0^t q(u) F(t-u)_{x=0} du + \frac{A_{f_2} \int_0^t q(u) F(t-u)_{x=0} \left[\int_0^u q(u') F(u-u')_{x=0} du' \right] du}{1 - A_{f_2} \int_0^t q(u) F(t-u)_{x=0} du} \quad (37)$$

2.2 Computations:

A two layer system Silicon-Glass is considered. The thickness of the front layer is 5 μm .

The physical properties of the materials chosen are given in the table (1).

Table 1 :
Physical and optical properties of the chosen material [41, 42,43]

Material	ρ (kgm^{-3})	λ ($\text{Wm}^{-1}\text{K}^{-1}$)	α (m^2s^{-1})	C_p ($\text{Jkg}^{-1}\text{K}^{-1}$)	A1	A_2 K^{-1}	T_m, K
Silicon	2.328×10^3	1.5×10^2	9.2×10^{-5}	700	0.678	3.12×10^{-5}	1683
Glass	2707	0.76	0.035×10^{-5}	0.8×10^3	-	-	-

The heat transfer coefficient " h_o " of the front surface is $1000 \text{ W/m}^2\text{K}$

The characteristics of the laser pulse are:

$$q_{\max} = 0.94 \times 10^9 \text{ W/m}^2,$$

$$t_o = 10 \mu\text{s}, t_d = 40 \mu\text{s}, m = 3$$

The thickness of the thin film $d = 10$ microns.

The obtained results for the function $\theta(o, t)$, K in the Silicon layer is computed for the considered case in which the absorptance is temperature dependent where $A_1 = 0.678$ and $A_2 = 3.12 \times 10^{-5} \text{ K}^{-1}$ [12].

The obtained results are illustrated graphically in Fig. 2.

The results reveal that the critical time required to initiate melting in the silicon zone is $t_m = 33.6 \mu\text{s}$.

Computations for the function $\theta_f(x, t)$ at $t = 34.4 \mu\text{s}$, revealed that the thermal penetration depth is $5 \mu\text{m}$.

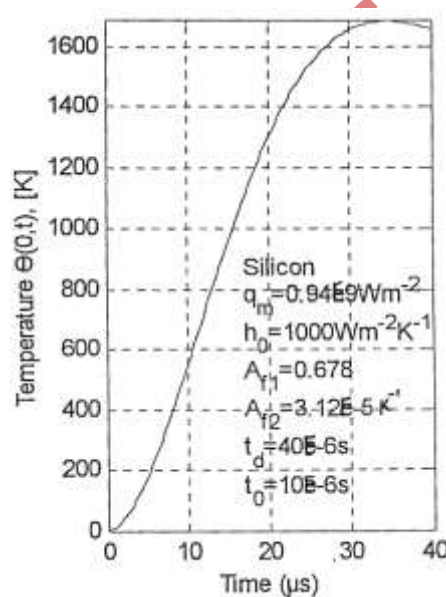


Fig. (1) : The function $\theta_f(o, t)$ for the Silicon layer in the two layer Silicon-Glass system

3. The effect of the laser pulse parameters on the thermal response.

In the following the effect of the laser pulse parameters on the thermal response of the irradiated two-layer system is studied, namely.

3.1 The effect of the maximum power density q_{\max} .

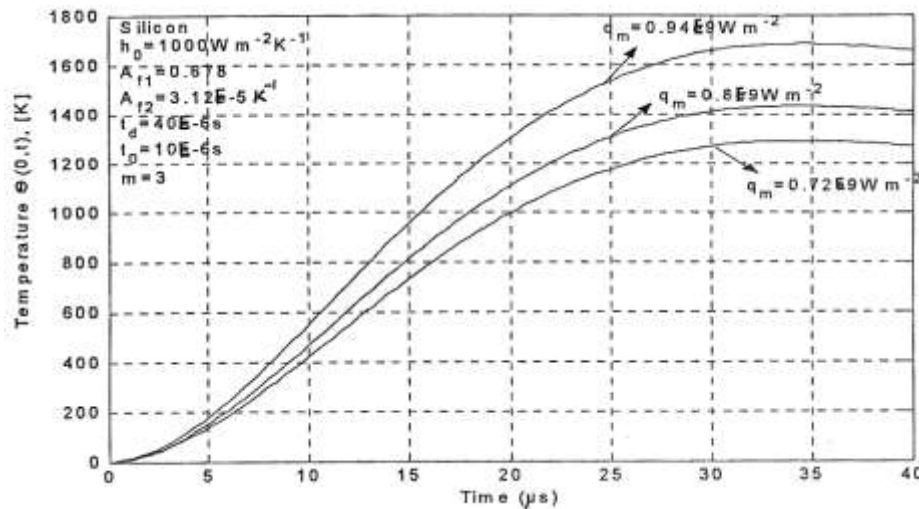


Fig. (2) : The function $\theta_f(o,t)$ at different values of q_{\max} for the chosen laser pulses two layer Silicon-Glass is considered

As a result it is revealed that higher levels of temperature are obtained in the irradiated solid target on increasing the maximum values of the incident laser power density.

Thus as q_{\max} increases, smaller values for the critical time " t_m " required to initiate melting are obtained.

3.2 The effect of " t_0 " on the thermal response of the irradiated target.

The time " t_0 " is the time interval characterizing the pulse at which the power reaches its maximum value q_{\max} .

Three values are considered:

$t_0 = 5, 7.5$ and $10 \mu s$.

The other parameters characterizing the laser pulse are the same.

The functions $\theta_f(o,t)$ are computed.

The obtained results are illustrated graphically in figure (3).

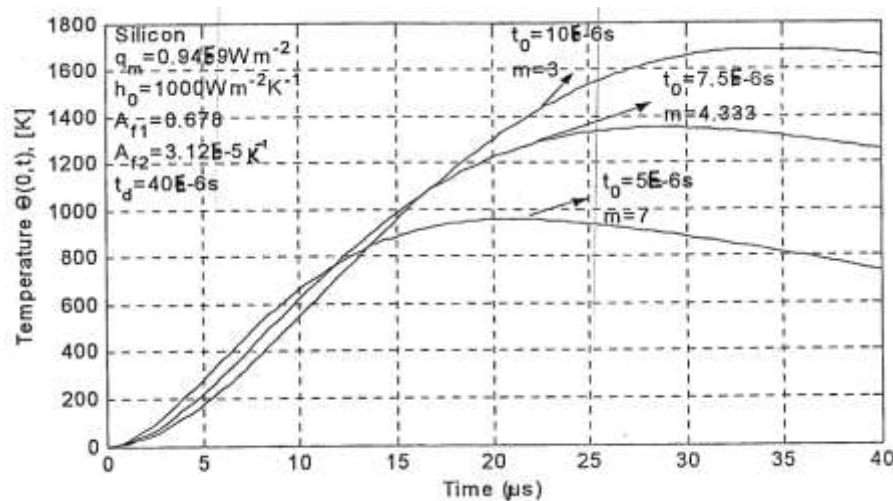


Fig. (3) : The function $\theta_f(o,t)$ at different values of " t_o " for f the chosen laser pulses .Two layer silicon-glass system is considered.

The parameter " t_o " is also related to the parameter " m " which in turn determines the general shape of the laser pulse.

This shape controls the total energy within one pulse consumed in the heating problem. It also controls the rate of heating during the growth of the pulse.

3.3 The effect of " t_d ", the pulse time duration on the thermal response of the two layer Silicon-Glass system.

Three values are considered in computing the function $\theta_f(o,t)$, K these values are :

$t_d = 35 \mu s$, $40 \mu s$, and $45 \mu s$, respectively.

The obtained results are illustrated graphically in Fig. (4).

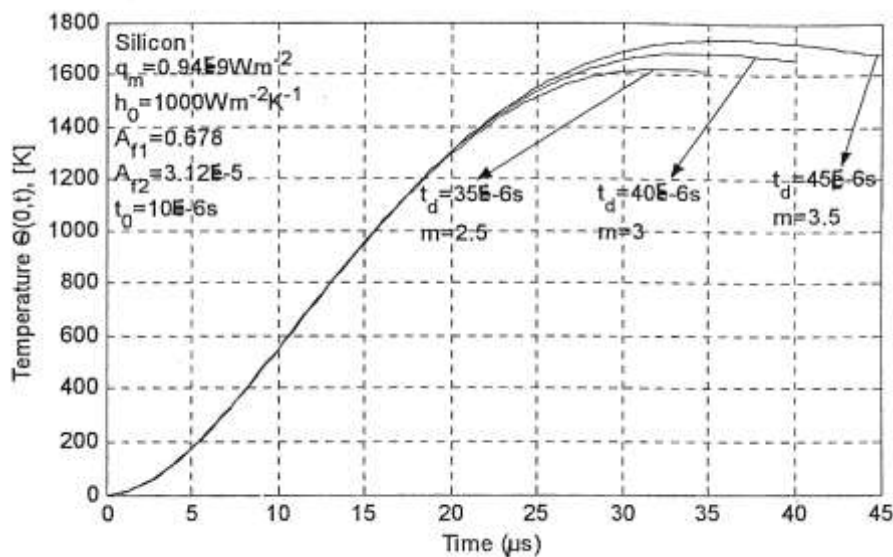


Fig. (4) : The function $\theta_f(o,t)$ at different values of the pulse time duration " t_d ". Two layer Silicon-Glass system is considered.

As a result it is revealed that the pulse time duration " t_d " is less effective. The curves for different pulses with different pulse time duration are nearly coincident as shown in Fig. (4). The other related operating conditions are the same.

This parameter does not also change principally the thermal penetration depth or the critical time required to initiate melting as revealed from other unpublished data.

Moreover, the dependence of the temperature dependent absorption coefficient is no longer linear. See equations (34) and (35).

Neglecting such dependence gives underestimated values for the critical time required to initiate melting or the values of the thermal penetration depth.

4.Conclusion :

1. The thermal response of the irradiated targets induced by laser heating is a function of the laser pulse parameters and the operating conditions.
2. The pulse parameters of predominant effect on heating are " q_{\max} " and " t_0 " while the pulse time duration is less effective.
3. The temperature dependence of the absorption coefficient at the front surface of the irradiated targets has to be taken into consideration otherwise underestimated values for the thermal responses are obtained.

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