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Exact Traveling Wave Solutions to Vakhnenko-Parkes Equation

Authors: Harun-Or-Roshid[2,*]; Md. Hafiz Uddin[4]; Mohammad Mobarak Hossain[1,4] ; Md. Hafizur Rahman[2]

[1]Department of Mathematics, Dhaka University of Engineering and Technology, Bangladesh
[2]Department of Mathematics, Pabna University of Science and Technology, Bangladesh
[3]Department of Mathematics, Jessore University of Science and Technology, Bangladesh
[4]Department of Mathematics, Hamdad University Bangladesh, Bangladesh

ABSTRACT

In this paper, we investigate some new traveling wave solutions to Vakhnenko-Parkes equation via improved (G'/G) *-expansion method. The solutions have been obtained including, periodic and solitons solutions interms of trigonometric, hyperbolic, and rational function solutions. The method is effective, efficient and applicable mathematical tools for nonlinear evolution equations (NLEEs).*

General Terms

Math. Subject Classification : 35K99, 35P05, 35P99.

Keywords

Vakhnenko- Parkes equation; improved (G'/G) **-expansion; traveling wave solutions.**

1. INTRODUCTION

Most of the phenomena in physics and other fields such as fluid mechanics, aero dynamics, nonlinear optics, plasma physics, hydrodynamics, chemistry, biology etc. are described by nonlinear partial differential equations (NLPDEs). In recent years, both mathematicians and physicist have devoted considerable effort to study of exact solution of the nonlinear ordinary or partial differential equation and many powerful methods have been presented. Finding traveling wave solutions is going research nonlinear for NLPDEs recently. For instance the inverse scattering transform [1], the complex hyperbolic function method [2, 3], the rank analysis method [4], the ansatz method [5, 6], the (G'/G)-expansion method [7-12], the modified simple equation method [13, 14], the exp-functions method[15], the sine-cosine method [16], the Jacobi elliptic function expansion method [17, 18], the F-expansion method[19, 20], the Backlund transformation method [21], the Darboux transformation method [22], the homogeneous balance method [23-25], the Adomian decomposition method [26], the auxiliary equation method[27], the $\exp(-\varphi(\xi))$-expansion method [28, 30] and so on. Recently, an interesting and important discovery has been made by Vakhnenko and Parkes [32], who have demonstrated that the reduced Ostrovsky equation [33]

$$(u_t + c_0 u_x + \alpha u u_x)_x = \gamma u$$

can be transformed to the new integrable equation

$$uu_{xxt} - u_x u_{xt} + u^2 u_t = 0 \qquad (1)$$

The traveling wave solutions Vakhnenko- Parkes equation was investigated in [31, 34, 35, 36] and Liu [36] found traveling wave solutions of this equations by improved (G'/G)-expansion method with auxiliary equation $GG'' = AG^2 + BGG' + C(G')^2$ but he used positive value of suffices in the solutions.

In this paper, we investigate the traveling wave solutions of the Vakhnenko- Parkes equation (1) with auxiliary equation $G'' + \lambda G' + \mu G = 0$ and consider both the positive and negative value of suffices in the trial solutions.

2. DESCRIPTION OF THE IMPROVED (G′/G)-EXPANSION METHOD

Suppose that we have a NLEE for $U(x,t)$ in the form

$$P(U, U_x, U_t, U_{xx}, U_{xt}, U_{tt}, \ldots\ldots) = 0 \qquad (2)$$

where P is a polynomial in its arguments, which includes nonlinear terms and the highest order derivatives.

Step 1. The transformation

$$U(x,t) = u(\xi), \quad \xi = x - wt, \qquad (3)$$

permits us reducing Eq.(2) to an ODE for $u = u(\xi)$,

$$P(u, u', u'', \cdots\cdots) = 0 \qquad (4)$$

Step 2. Suppose that the solution of ODE (3) can be expressed by a polynomial in $('G/G)$ as follows

$$u=\sum_{i=-m}^{m} l_i (G'(\xi)/G(\xi))^i \qquad (5)$$

where $G(\xi)$ satisfies the ODE in

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0, \qquad (6)$$

then the solutions of ODE (6) are

When $\lambda^2-4\mu>0$, then

$$G'/G=\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)-\frac{\lambda}{2} \qquad (7)$$

When $\lambda^2-4\mu<0$, then

$$G'/G=\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right)-\frac{\lambda}{2} \qquad (8)$$

When $\lambda^2-4\mu=0, \mu=\lambda=0$, then $G'/G=\dfrac{C_1}{C_1\xi+C_2}$ (9)

$l_i, w, \lambda, \mu;\ i=-m,\cdots\cdots,m$ are constants to be determined later, l_m and l_{-m} are not both zero simultaneously. And the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (4). By substituting (5) into Eq.(4) and using the ODE (6), collecting all terms with the same order of $(G'/G)^i;\ i=-m,\cdots\cdots,m$ together, and setting them to zero, yields a set of algebraic equations for $l_i,\cdots w,\lambda;\ i=-m,\cdots\cdots,m$ and μ.

Now, solving the algebraic equations for $l_i,\cdots w,\lambda;\ i=-m,\cdots\cdots,m$ and μ and putting in the general solutions of ODE (6), we obtain the general solutions of Eq.(1).

3. APPLICATION

In this section, we will demonstrate the improved (G'/G)-expansion method on Vakhnenko- Parkes equation (1).

Substitute (3) into (1) we change Eq. (1) into the ODE:

$$uu'''-u'u''+u^2u'=0 \qquad (10)$$

Integrating Eq. (10) once with respect to ξ and setting the integration constant equal to zero yields

$$3uu''-3(u')^2+u^3=0 \qquad (11)$$

Balance the highest order derivate term uu'' and the highest nonlinear terms u^3 in equation (11), we get $m=2$, so assume the equation (1) has the following solution

$$u(\xi)=l_0+l_1(G'/G)+l_2(G'/G)^2+l_{-1}(G/G')+l_{-2}(G/G')^2 \qquad (12)$$

where $u(x,t)=u(\xi),\ \xi=x-wt$ and l_2 and l_{-2} are not both zero simultaneously.

Substitute (12) and (6) into (11), let the coefficient of $(G'/G)^i,\ (i=\cdots\cdots-2,1,0,1,2\cdots\cdots)$ be zero, yields a set of algebraic equations about l_i, w as follows:

For convenience we omitted the equations and solving them for $l_{-2}, l_{-1}, l_0, l_1, l_2$ by Maple 13, we achieve the following solutions:

Set-1: $l_{-2}=l_{-1}=0,\ l_0=-6\mu,\ l_1=-6\lambda,\ l_2=-6$ and

Set-2: $l_{-2}=-6\mu^2,\ l_{-1}=-6\lambda\mu,\ l_0=-6\mu,\ l_1=0,\ l_2=0$

For the set 1, we have the solutions

$$u(\xi)=-6\mu-6\lambda(G'/G)-6(G'/G)^2 \qquad (13)$$

When $\lambda^2-4\mu>0$, then

$$u(\xi)=-6\mu-6\lambda\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}{C_1\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}\right)-\frac{\lambda}{2}\right)$$
$$-6\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}{C_1\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}\right)-\frac{\lambda}{2}\right)^2$$

(14)

where $\xi=x-wt$ and C_1, C_2 and w are arbitrary constants.

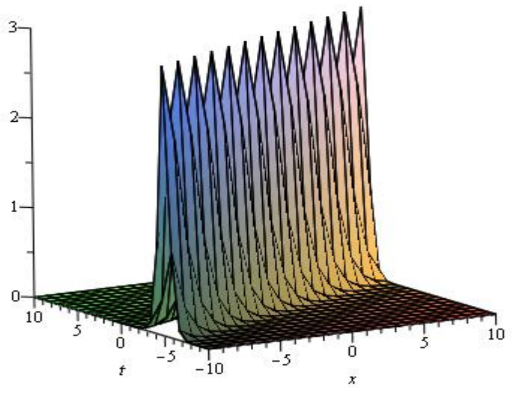

Fig 1. Shape of in 3D Eq. (14) for $C_1=\lambda=w=2,\ \mu=0.5, C_2=0$

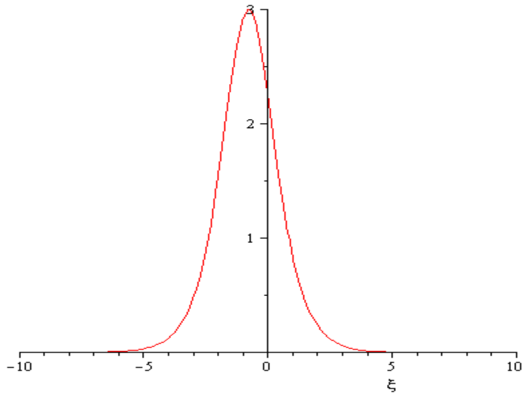

Fig 2. Shape of in 2D Eq. (14) for $C_1=\lambda=2$, $\mu=0.5, C_2=0$

When $\lambda^2-4\mu<0$, then

$$u(\xi)=-6\mu-6\lambda\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}{C_1\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}\right)-\frac{\lambda}{2}\right)$$
$$-6\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}{C_1\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}\right)-\frac{\lambda}{2}\right)^2 \quad (15)$$

where $\xi=x-wt$ and C_1, C_2 and w are arbitrary constants.

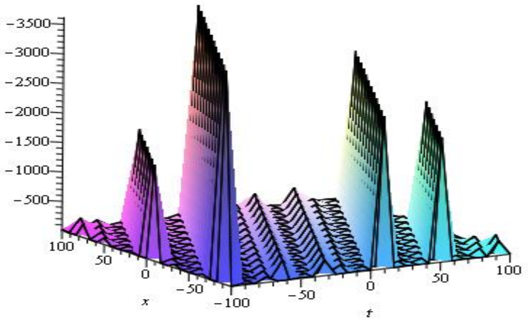

Fig 3. Shape of in 3D Eq. (15) for $C_1=\lambda=1$, $\mu=1, w=C_2=2$

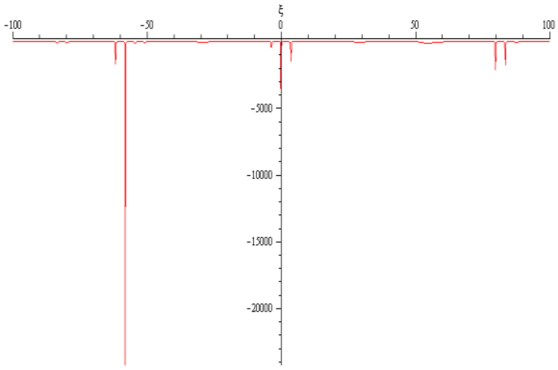

Fig 4. Shape of in 2D Eq. (15) for $C_1=\lambda=1$, $\mu=1, C_2=2$

When $\lambda^2-4\mu=0, \mu=\lambda=0$, then

$$u(\xi)=-6\left(\frac{C_1}{C_1\xi+C_2}\right)^2 \quad (16)$$

where $\xi=x-wt$ and C_1, C_2 and w are arbitrary constants. Shape of solution (16) is similar to the shape of Figures of solutions (14).

For the set 2, we have the solutions

$$u(\xi)=-6\mu-6\lambda\mu\,(G'/G)^{-1}-6\mu^2\,(G'/G)^{-2} \quad (17)$$

When $\lambda^2-4\mu>0$, then

$$u(\xi)=-6\mu-6\lambda\mu\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}{C_1\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}\right)-\frac{\lambda}{2}\right)^{-1}$$
$$-6\mu^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}{C_1\cosh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)+C_2\sinh\frac{1}{2}\left(\sqrt{\lambda^2-4\mu\xi}\right)}\right)-\frac{\lambda}{2}\right)^{-2} \quad (18)$$

where $\xi=x-wt$ and C_1, C_2 and w are arbitrary constants. Shape of solution (18) is similar to the shape of Figures of solutions (14).

When $\lambda^2-4\mu<0$, then

$$u(\xi)=-6\mu-6\lambda\mu\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}{C_1\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}\right)-\frac{\lambda}{2}\right)^{-1}$$
$$-6\mu^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}{C_1\cos\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)+C_2\sin\frac{1}{2}\left(\sqrt{4\mu-\lambda^2}\,\xi\right)}\right)-\frac{\lambda}{2}\right)^{-2} \quad (19)$$

where $\xi=x-wt$ and C_1, C_2 and w are arbitrary constants. Shape of solution (19) is similar to the shape of Figures of solutions (15).

When $\lambda^2-4\mu=0, \mu=\lambda=0$, then the solution is trivial.

Remark: All of the solutions presented in this latter have been checked with Maple by putting them back into the original equations.

4. CONCLUTION AND DISCUSSION

This paper, we get new exact traveling wave solutions for the Vakhnenko- Parkes equation, including the hyperbolic functions, trigonometric functions and rational function solutions. To the best of our knowledge, this equation is not solved via the improved (G'/ G) -expansion method with the auxiliary equation $G''+\lambda G'+\mu G=0$ considering both the positive and negative value of suffices in the trial solutions. Many researchers solved the Vakhnenko- Parkes equation for obtaining analytical solutions by using different methods. For instance, Kangalgil and Ayaz [31] studied this equation by applying the auxiliary equation method to obtain exact solutions. Abazari [35] investigated this equation via this basic (G'/ G) -expansion method to construct traveling wave solutions and By basic method only solutions Eq. (14) and Eq. (15) are obtained. This equation is investigated by Yasar [34] via improved tanh method to establish analytical solutions of the same equation while Liu [36] found traveling wave solutions of this equations by improved (G'/G)-expansion method with auxiliary equation $GG''=AG^2+BGG'+C(G')^2$ but he used positive value of suffices in the solutions. Liu [36] obtained few different solutions including the solutions obtained by basic (G'/G)-expansion method. We have studied mentioned equation by applying the improved (G'/ G) -expansion method and abundant solutions are constructed in this article. So, other three solutions Eq. (16), Eq. (18) and Eq. (19) are new. Furthermore, the improved (G'/G) method appears to be easier, faster and can be handle by computer easily and we used Maple-13 to solve the equation. This will have a good sense to promote the extensive application of the equations.

5. ACKNOWLEDGMENTS

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