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A New Stability Criterion For Fixed-Point Digital Filters With Saturation Arithmetic In Presence Of External Disturbance

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ABSTRACT

A new exponential stability and H∞ performance of fixed–point digital filter under saturation overflow nonlinearity and external disturbance is presented. Compared with Ahn CK [1] and Kokil P, Kandanvli VKR, Kar H. [2] the present criterion is lax and can include the existing result as a special case. A numeric example is presented which proves the proficiency of the proposed criterion.

Keywords:

H∞ approach, Exponential stability, Digital filters, Finite Word length effect, Linear Matrix Inequality

1. Introduction

Digital filters play an important role in Digital signal
in [2].The relation (9) exploits the structural properties of the multiple saturation nonlinearities (i.e. the passivity property [6, 8].Note that $\phi(r)$ is non-positive in view of (3), if the LMI (5) is satisfied, we have

$$\Delta V(x(r)) < -x^T(r)Sx(r) + \gamma^2 w^T(r)w(r) \qquad (10)$$

Summing both sides of (10) from 0 to ∞ we have

$$V(x(\infty)) - V(x(0)) < -\sum_{r=0}^{\infty} x^T(r)Sx(r) + \gamma^2 \sum_{r=0}^{\infty} w^T(r)w(r)$$

processing. So these filters must be implemented on a digital computer or on a special-purpose digital hardware, but while implementing them, nonlinearities such as quantization and overflow due to finite word length occurs, this results in instability (zero input limit cycle and overflow limit cycle) of the otherwise stable digital filters. The quantization and overflow nonlinearities may interact with each other. However, if the number of quantization steps is large, then the effect of these nonlinearities can be regarded as decoupled or noninteracting and can be investigated separately. The stability property of fixed-point digital filter employing saturation overflows nonlinearity have been studied by several researchers [3-8].However, most existing stability criteria for digital filters are only available under zero input condition, while in presence of external disturbance and zero input , most existing criterion are of no use.

The purpose of this paper is to develop a new stability criterion for digital filters under unfavourable conditions i.e. in presence of external disturbance so that it can overcome the short comings of most of the existing stability criteria for digital filters.

2. System description

The system under consideration is described by:

$$x(r+1) = f(y(r)) + w(r) = [f_1(y_1(r)) \quad f_2(y_2(r)).......f_n(y_n(r))]^T$$
$$+[w_1(r) \quad w_2(r).....w_n(r)]^T \qquad (1)$$

$$y(r) = [y_1(r) \quad y_2(r).....y_n(r)]^T = Ax(r) \qquad (2)$$

Where $x(r) \in R^n$ is a state vector, $w(r) \in R^n$ is an external disturbance, $A \in R^{n \times n}$ is the coefficient matrix, and T denotes the transpose The following saturation nonlinearities :

$$f_i(y_i(r)) = \begin{cases} 1 & y_i(r) > 1 \\ y_i(r) & |y_i(r)| \le 1 \\ -1 & y_i(r) < -1 \end{cases} \qquad (3)$$

are under consideration for i =1,2,.......n.

For a given $\gamma > 0$, the purpose of this paper is to develop a new LMI criterion such that the system (1) and (2) with $w(r) = 0$ is exponentially stable and

$$\sum_{r=0}^{\infty} x^T(r)Sx(r) < \gamma^2 \sum_{r=0}^{\infty} w^T(r)w(r) \qquad (4)$$

under zero-initial conditions for all nonzero $w(r)$, where S is a positive symmetric matrix. The parameter γ is called the H_∞ norm bound or the disturbance attenuation level. In this, case the system (1) and (2) is said to be exponentially stable with H_∞ performance γ .

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3. New LMI criterion

The new exponential stability criterion is given in the following theorem

Theorem1. For a given $\gamma > 0$, if there exist symmetric positive definite matrices P, S and positive definite diagonal dominant matrices H and C such that

$$\begin{bmatrix} A^T HA + S - P & A^T C & 0 \\ C^T A & P - C - C^T - H & P \\ 0 & P & P - \gamma^2 I \end{bmatrix} < 0 \qquad (5)$$

Then the system (1)-(3) is exponentially stable with H_∞ performance γ.

Proof: To establish the H_∞ performance for the system (1)-(3), considered the following Lyapunov function:

$$V(x(r)) = x^T(r)Px(r) \qquad (6)$$

Along the trajectory of (1), we have

$$\begin{aligned} \Delta V(x(r)) &= V(x(r+1)) - V(x(r)) \\ &= [f(Ax(r) + w(r)]^T P[f(Ax(r)) + w(r)] - x^T(r)Px(r) \\ &= f^T(Ax(r))Pf(Ax(r)) + f^T(Ax(r))Pw(r) \\ &\quad + w^T(r)Pf(Ax(r)) + w^T(r)Pw(r) - x^T(r)Px(r) \end{aligned}$$

Then according to ref [4], if H is a positive definite diagonal dominant matrix, then

$$y^T(r)Hy(r) - f^T(y(r))Hf(y(r)) \ge 0 \qquad (7)$$

Using (2) and (7), we have

$$x^T(r)A^T HAx(r) - f^T(Ax(r))Hf(Ax(r)) \ge 0 \qquad (8)$$

Using (8), a new bound for $\Delta V(x(r))$ can be obtained as

$$\begin{aligned} \Delta V(x(r)) &\le f^T(Ax(r))Pf(Ax(r)) + f^T(Ax(r))Pw(r) \\ &\quad + w^T(r)Pf(Ax(r)) + w^T(r)Pw(r) - x^T(r)Px(r) \\ &\quad + x^T(r)A^T HAx(r) - f^T(Ax(r))Hf(Ax(r)) \end{aligned}$$

$$= \begin{bmatrix} x(r) \\ f(Ax(r)) \\ w(r) \end{bmatrix}^T \begin{bmatrix} A^T HA - P + S & A^T C & 0 \\ C^T A & P - C - C^T - H & P \\ 0 & P & P - \gamma^2 I \end{bmatrix} \times \begin{bmatrix} x(r) \\ f(Ax(r)) \\ w(r) \end{bmatrix} - x^T(r)Sx(r) + \gamma^2 w^T(r)w(r) + \phi(r)$$

Where $\phi(r) = -y^T(r)Cf(y(r)) - f^T(y(r))C^T y(r)$

$$+ f^T(y(r))(C + C^T)f(y(r)) < 0 \qquad (9)$$

Where C is a positive definite diagonal dominant matrix, which has been utilized

since $V(x(\infty)) \ge 0$ and $V(x(0)) = 0$,we have the relation(4)

Following [1] we will prove that, under LMI condition (5), the system (1) and (2) with $w(r) = 0$ is exponentially stable. Satisfies following Rayleigh inequality [9]:

$$\lambda_{\min}(P)\|x(r)\|^2 \le V(x(r)) \le \lambda_{\max}(P)\|x(r)\|^2 \qquad (11)$$

Where $\lambda_{\max}$ (.) and $\lambda_{\min}$ (.) are the maximum and minimum eigen values of the matrix respectively. When $w(r) = 0$, we have

$$\Delta V(x(r)) < -x^T(r)Sx(r) \le -\lambda_{\min}(S)\|x(r)\|^2 \qquad (12)$$

from (10), according to Theorem 3.1 of [10],(11) and (12) guarantees the exponential stability of the system under consideration. This completes the proof.

As discussed in [1, Remark2], the relation (4) can be represented by H (∞) <γ^2 where

$$H(r) = \frac{\sum_{k=0}^{r} x^T(k)Sx(k)}{\sum_{k=0}^{r} w^T(k)w(k)} \qquad (13)$$

Remark 1: Theorem 2 [2] is a special case of Theorem 1 when,

$$H = \delta I \qquad (14)$$

Where δ is a positive scalar.

Remark 2: To check the feasibility of (5) for a given γ and A, following [2] we will choose the matrices C and H as, $C = [c_{ij}] \in R^{n\times n}$ and $H = [h_{ij}] \in R^{n\times n}$ respectively in the following form:

$$c_{ii} = \sum_{j=1, j\ne i}^{n} (\alpha_{ij} + \beta_{ij}) \qquad i=1, 2 \ldots n \qquad (15a)$$

$$c_{ij} = \alpha_{ij} - \beta_{ij} \qquad i, j=1,2,\ldots n (i\ne j) \qquad (15b)$$

$$h_{ii} = \sum_{j=1, j\ne i}^{n} (\eta_{ij} + \mu_{ij}) \qquad i=1,2\ldots n \qquad (15c)$$

$$h_{ij} = \eta_{ij} - \mu_{ij} \qquad i, j=1,2,\ldots n (i\ne j) \qquad (15d)$$

$$\alpha_{ij} > 0, \beta_{ij} > 0, \eta_{ij} > 0, \mu_{ij} > 0 \quad (15e)$$

The matrices C and H described by (15) will be positive definite diagonal dominant matrices, and with the matrices C and H given by (15), the matrix inequality (5) becomes linear in the unknown parameters $\alpha_{ij}s, \beta_{ij}s, \eta_{ij}s, \mu_{ij}s$, P and S. Therefore, (5) can be solved efficiently by employing the MATLAB LMI toolbox [11-12].

4. Numerical examples

Example 1 Consider a second- order system (1)-(3) with

$$A = \begin{bmatrix} 1.1 & -7.4 \\ 0.12 & 0 \end{bmatrix}$$

For the design objective (4), let the H∞ performance be specified by γ=0.45, using MATLAB LMI tool box [11-12], it is verified that Theorem 2 [2] fails to prove the exponential stability of this example and does not provide any feasible solution for the above example.

Now we will apply Theorem 1 in the example under consideration In view of Remark 2, to check the feasibility of (5), we choose C and H matrices as

$$C = \begin{bmatrix} \alpha_{12}+\beta_{12} & \alpha_{12}-\beta_{12} \\ \alpha_{21}-\beta_{21} & \alpha_{21}+\beta_{21} \end{bmatrix} \quad H = \begin{bmatrix} \eta_{12}+\mu_{12} & \eta_{12}-\mu_{12} \\ \eta_{21}-\mu_{21} & \eta_{21}+\mu_{21} \end{bmatrix}$$

Where $\alpha_{12} > 0$, $\beta_{12} > 0$, $\alpha_{21} > 0$, $\beta_{21} > 0$, $\eta_{12} > 0$, $\mu_{12} > 0$, $\eta_{21} > 0$, and $\mu_{21} > 0$.With the help of MATLAB LMI tool box [11-12] ,it found that (5) is feasible for the example under consideration with following values of unknown parameters:

$$P = \begin{bmatrix} 0.0003 & -0.0020 \\ -0.0020 & 0.0165 \end{bmatrix} \quad S = \begin{bmatrix} 0.000002 & -0.000009 \\ -0.000009 & 0.0002 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.00006 & -0.00003 \\ -0.0017 & 0.0073 \end{bmatrix} \quad C = \begin{bmatrix} 0.0002 & -0.0002 \\ -0.0004 & 0.0106 \end{bmatrix}$$

($\alpha_{12}=2.5433\times10^{-5}$, $\beta_{12}=1.9239\times10^{-4}$, $\alpha_{21}=0.0051$, $\beta_{21}=0.0055$, $\eta_{12}=1.7536\times10^{-5}$, $\mu_{12}=4.5987\times10^{-5}$, $\eta_{21}=0.0028$, $\mu_{21}=0.0045$).

Therefore Theorem 1 is an improved LMI criterion as compared with Theorem 2 of [2].

Fig.1 shows the plot of $H(r)$ and verifies H (∞) <γ^2=0.2025
This means that the H∞ norm from the external disturbance $w(r)$ to the state vector $x(r)$ is reduced within the H∞ norm bound γ

Example 2. As per the example 2 considered in [2] with

$$A = \begin{bmatrix} 0.99 & -2 \\ 0.1 & 0 \end{bmatrix} \qquad w(r) = \begin{bmatrix} \cos(r) \\ \sin(r) \end{bmatrix}$$

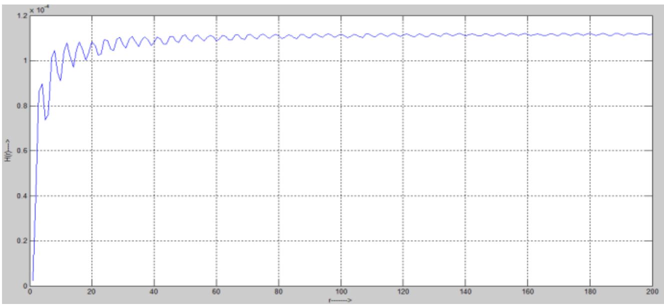

Fig1. The plot of H(r)

The minimum lower bound of γ for the above system as obtained via Theorem 1[2] and Theorem 2[2] was 148 $\times10^{-5}$ and 8.7×10^{-5} respectively. While applying Theorem 1 in this case yields minimum lower bound of γ as 6.6 $\times10^{-5}$.The minimum lower bound obtained via Theorem 1 is lowest, rather Theorem 1 provides improved results for the example under consideration.

5. Conclusion

In this paper a new and highly relaxed criterion for the exponential stability and H∞ performance for fixed-point digital filter under saturation nonlinearity and external disturbances has been presented.

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