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Performance Analysis of OFDM with Wiener Phase Noise

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Abstract*— We analyze the effect of Wiener phase noise on the capacity and signal-to-interference-plus-noise (SINR) ratio. Our analysis includes phase noise at the transmitter and receiver end of an OFDM communication link. We see that the capacity and SINR are random variables whose distribution depends on the phase noise and the fading channel. Using a Taylor series approximation, we show that the random variable, characterizing the phase noise, in the performance metrics can be expressed as a sum of correlated gamma variables with rank-deficient square-root normalized covariance matrix. The approximation holds well when the ratio between the subcarrier spacing and the 3dB bandwidth of the oscillator power spectral density is atleast one order of magnitude, which for most practical oscillators is the case. In earlier literature, the probability density function of a sum of correlated gamma variables with full-rank square root normalized covariance matrix was derived. We extend these results to the rank-deficient case and apply them to the random variable of our case. With the probability density functions characterizing the phase noise and the fading channel at hand, we proceed to obtain closed-form statistical measures of capacity and SINR. The simulations show the good agreement with our analytical expressions.

***Index Terms*—Phase noise, orthogonal frequency division multiplexing, gamma variables, capacity, intercarrier interference.**

I. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) is a multi-carrier modulation technique that has been, or is in the process of being, incorporated in numerous standards. However, the reliability of systems and services, that are an outcome of these standards, is heavily lessened by RF impairments that occur at the analog frontend of a communication system. Impairments such as frequency offset, phase noise, IQ imbalance, power amplifier non linearities have been extensively studied. Our focus in this paper shall be the effect of phase noise on OFDM systems. Phase noise (PN) is the random fluctuation in the phase of the sinusoid waveform used in the up and down conversion processes of signals between baseband and RF. These occur due to imperfections in the oscillators.

With respect to OFDM, PN manifests itself in the form of a common phase error (CPE) and intercarrier interference (ICI). The CPE causes a rotation of the signal constellation while the ICI, being additive, is more noise-like. In earlier literature, performance analysis of OFDM systems with PN have been in terms of signal to interference-plus-noise ratio

(SINR) and bit-error rates (BER). To the best of our knowledge, except for, there is no literature related to the capacity of OFDM systems impaired by PN. In, the capacity is derived considering not only PN but also transmitter nonlinearities, channel estimation errors and frequency selective channels. However, the capacity is not obtained in closed form and needs to be evaluated numerically. In contrast, we choose capacity and SINR as performance metrics for which closed form statistical expressions are obtained.

In this paper, we adopt a probability density function (PDF)
approach for evaluating the average SINR and capacity, the rationale for which is as follows. For one OFDM symbol we get one realization of the PN process and of the channel. Then assuming (independent identically distributed) Gaussian codebook, the SINR, conditioned on this one realization, depends only on that particular realization of PN process and the channel. Thus, we see that the SINR is a random variable
(RV) as different OFDM symbols yield different realizations
of the PN process and of the channel. Thus, knowledge of the
PDF of the SINR provides an accurate estimate of its average.
We can use the same argument for evaluating the capacity. The receiver noise along with the ICI will be

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Gaussian conditioned on one realization of the PN process and the channel. Hence, as the SINR is a RV for different PN and channel realizations, so will the capacity.

II. EXISTING METHODS

Previous approaches of evaluating the average SINR have been toward obtaining accurate statistics of the CPE and ICI. In [22], the SINR was derived using the small angle approximation for the PN while the approach in [23] was to use the power spectral density (PSD) of the PN. SINR bounds showing its dependence on various system parameters, was derived in [24] using a linear approximation of the PN. The work in [25] focused on a nonlinear approximation of the PN,
and the work in [26] generalizes the approach for any PN level and any number of subcarriers. SINR expressions for three different types of receivers was considered in [27]. A detailed analysis of the statistical properties of the discrete Fourier transform (DFT) of the PN process is considered in [28] and is used in a minimum mean square error based ICI suppression algorithm. SINR expressions under carrier frequency offset, PN and timing jitter over a Rayleigh fading channel were derived in [10]

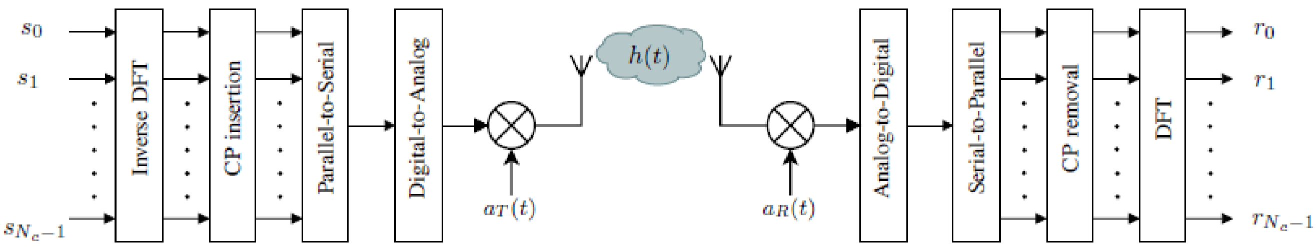

Figure 1:OFDM impaired with phase noise

III. PROPOSED SYSTEM

A.System Model

A typical OFDM system with N_c subcarriers is considered. The input symbols $\{s_j\}_{j=0}^{N_c-1}$ are converted to the discrete time domain by the inverse discrete Fourier transform (IDFT) operation. Cyclic prefix is added to combat intersymbol interference followed by the parallel-to-serial and digital-to-analog conversion to obtain the baseband signal.This baseband signal is converted to RF by the transmit oscillator $a_T(t) = e^{j(2\pi f_c t + \theta_T(t))}$ with PN $\theta_T(t)$ and carrier frequency f_c .The transmitted RF signal passes through the channel *h(t)* and at the receiver, it is converted back to baseband by the receive oscillator $a_R(t) = e^{-j(2\pi f_c t - \theta_R(t))}$ where $\theta_R(t)$ is the receiver PN. The received baseband signal is converted to the discrete frequency transmit domain by applying sequentially the analog-to-digital, serial-to-parallel and DFT operations to obtain $\{r_j\}_{j=0}^{N_c-1}$. Following this signal path, we arrive at the expression for the received symbols as

$$r_j = \left(\sum_{i=0}^{N_c-1} H_i \delta_{i-j}^R \delta_{-i+j}^T\right) s_j + \sum_{k=0,k\neq j}^{N_c-1} \left(\sum_{i=0}^{N_c-1} H_i \delta_{i-j}^R \delta_{-i+k}^T\right) s_k + n_j \quad (1)$$

Since

$$\delta_i^x = \frac{1}{N_c} \sum_{n=m(N_c+N_{cp})}^{m(N_c+N_{cp})+N_c-1} e^{j\theta x(n)} e^{-j2\pi i n / N_c}$$

We can write δ_{i-j} as,

$$\delta_{i-j} = \frac{1}{N_c} \sum_{n=m(N_c+N_{cp})}^{m(N_c+N_{cp})+N_c-1} e^{j\left(\theta^{T(n)}+\theta^{R(n)}\right)} e^{-j(2\pi(i-j)n)/N_c} \quad (2)$$

Since $f_{3dB} < f_{sub}$ δ_{i-j}^R is maximum at i=j

$$\delta_{i-j}^R H_i \approx \begin{cases} 0 \\ \delta_{i-j}^R \end{cases}$$ For large i-j and for small i-j

Approximate weights,

$$\left(\delta_{i-j}^R H_i\right) * \delta_i^T \approx H_j\left(\delta_{i-j}^R * \delta_i^T\right) \approx H_j \delta_{i-j}$$

(3)

Substitute (3) in (1)

$$r_j = H_j \delta_{i-j} \delta_{-i+j} s_j + \sum_{k=0,k\neq j}^{N_c-1} H_j \delta_{i-j} \delta_{-i+k} s_k + n_j$$

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$$\therefore r_j = H_j\delta_0 s_j + \sum_{k=0,k\neq j}^{N_c-1} H_j\delta_{k-j}s_k + n_j \quad \textbf{(4)}$$

B.Signal-to-Interference-Plus-Noise-Ratio

In order to evaluate the SINR per subcarrier, we assume first that the input symbols $\{s_j\}_{j=0}^{N_c-1}$ are independent of each other. The noise n_j is also assumed independent of the input symbols as well as of the PN. The channel coherence time is typically larger compared to the OFDM symbol length, and thus, H_j *would* be constant over the symbol length.

Squaring equation (4),

$$|r_j|^2 = |H_j|^2|\delta_0|^2|s_j|^2 + \left|\sum_{k=0,k\neq j}^{N_c-1}|H_j|^2|\delta_{k-j}|^2|s_k|\right|^2 + |n_j|^2$$

We keep H_j and δ_k as constant and take expectation for the other terms,

$$\mathrm{E}\left[|r_j|^2\right] = |H_j|^2|\delta_0|^2\mathrm{E}\left[|s_j|^2\right] + \sum_{k=0,k\neq j}^{N_c-1}|H_j|^2|\delta_{k-j}|^2\mathrm{E}\left[|s_k|^2\right] + \mathrm{E}\left[|n_j|^2\right]$$

we know that $\mathrm{E}\left[|s_j|^2\right] = \sigma_s^{\,2}$ and $\mathrm{E}\left[|n_j|^2\right] = \sigma_n^{\,2}$,hence

$$\mathrm{E}\left[|r_j|^2\right] = |H_j|^2|\delta_0|^2\sigma_s^{\,2} + \sum_{k=0,k\neq j}^{N_c-1}|H_j|^2|\delta_{k-j}|^2\sigma_s^{\,2} + \sigma_n^{\,2} \quad (6)$$

From equation (6) we get SINR as

$$\gamma_j = \frac{|H_j|^2|\delta_0|^2\sigma_s^{\,2}}{\sum_{k=0,k\neq j}^{N_c-1}|H_j|^2|\delta_{k-j}|^2\sigma_s^{\,2} + \sigma_n^{\,2}} \quad (7)$$

We denote $y = \sum_{k=1}^{N_c-1}|\delta_k|^2$ and $g_j = |H_j|^2$

(8)

By parseval's theorem,

$$|\delta_0|^2 = 1 - \sum_{k=1}^{N_c-1}|\delta_k|^2$$

$$\therefore |\delta_0|^2 = 1 - y \quad (9)$$

Substitute (9) and (8) in (7) we get,

$$\gamma_j = \frac{g_j(1-y)\sigma_s^{\,2}}{g_j y\sigma_s^{\,2} + \sigma_n^{\,2}} = \frac{g_j\sigma_s^{\,2}(1-y)}{g_j\sigma_s^{\,2}\left[y + \frac{\sigma_n^2}{g_j\sigma_s^2}\right]}$$

$$\therefore \gamma_j = \frac{1-y}{y + \frac{\sigma_n^2}{g_j\sigma_s^2}} \quad \textbf{(10)}$$

Denoting the respective RVs of the realizations y *and* g_j by

Y and *Gj*, (10) shows how the SINR depends on the PN process (at the transmitter and receiver) and the channel. Because *Y* and *Gj* can be assumed independent of each other, the average SINR (or average of any function of the SINR) is obtained by sequentially averaging over the PDFs of *Y* and *Gj*. Assuming a Rayleigh fading channel, we now proceed to obtain the PDF

C. Weiner Phase Noise

For autonomous oscillators, as $t \to \infty$, the PN $\theta(t)$ becomes asymptotically a Gaussian process with variance $\sigma^2 = ct$ that linearly increases with time t=T_s and c being the rate of the variance whose value depends on the kind of oscillator used. We can describe such a process as being a Wiener process or Brownian motion. A discrete Wiener process $\theta(nTs)$ is obtained by sampling its continuous time counterpart $\theta(t)$. It is typically given as

$$\theta[n] = \sum_{i=0}^{n}\varepsilon(i)$$

By definition of weiner phase noise process $\theta(0) = \varepsilon(0) = 0$ and $\varepsilon(i) = \theta(i) - \theta(i-1)$ are independent and its variance is given as,

$$\sigma^2 = cT_s$$

We know that $T_s = \dfrac{1}{f_{sub}N_c}$

$$\sigma^2 = \frac{c}{f_{sub}N_c}$$

Since $c = 4\pi f_{3dB}$

$$\therefore \sigma^2 = \frac{4\pi f_{3dB}}{f_{sub}N_c} \quad \textbf{(11)}$$

D. Taylor Series Approximation

In order to evaluate the average SINR, average and outage capacity from (10), we need the PDF of *Y* which we show next, can be expressed as a sum of gamma RVs using the Taylor series approximation. Variance of weiner phase noise over one OFDM symbol is small and is given as,

$$\sigma_{\max}^2 = (N_c - 1)\sigma^2$$

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From (11) we get,

$$\sigma^2_{\max} = (N_c - 1)\frac{4\pi f_{3dB}}{f_{sub} N_c}$$

$N_c >> 1$

$$\sigma^2_{\max} = \frac{N_c 4\pi f_{3dB}}{f_{sub} N_c}$$

$$\therefore \sigma^2_{\max} \approx \frac{4\pi f_{3dB}}{f_{sub}} \qquad \textbf{(12)}$$

As long as the accumulated variance of the Wiener PN process over one OFDM symbol is sufficiently small, i.e., $\sigma^2_{\max} = (N_c - 1)\sigma^2$ y is a sum of $N = \frac{N(N_c - 1)}{2}$ correlated gamma variables as follows

$$y = \sum_{k=1}^{N_c - 1} |\delta_k|^2 \approx \sum_{l=1}^{N_c - 1} \sum_{i=1}^{N_c - l} Z_{il} \qquad \textbf{(13)}$$

where Z_{il} follows a gamma distribution with parameters $\alpha = 1/2$ and $\beta_l = \frac{2\sigma^2 l}{N_c^{\ 2}}$ and is given as

$$Z_{il} = \frac{1}{2}\beta_l \left(\frac{\sum_{j=0}^{l-1} \varepsilon\left(i + m\left(N_c + N_{cp}\right) + j\right)}{\sqrt{l\sigma}} \right)^2$$

IV. RESULTS AND DISCUSSIONS

In this section, the analytical performance measures derived in Section III are compared with simulations.

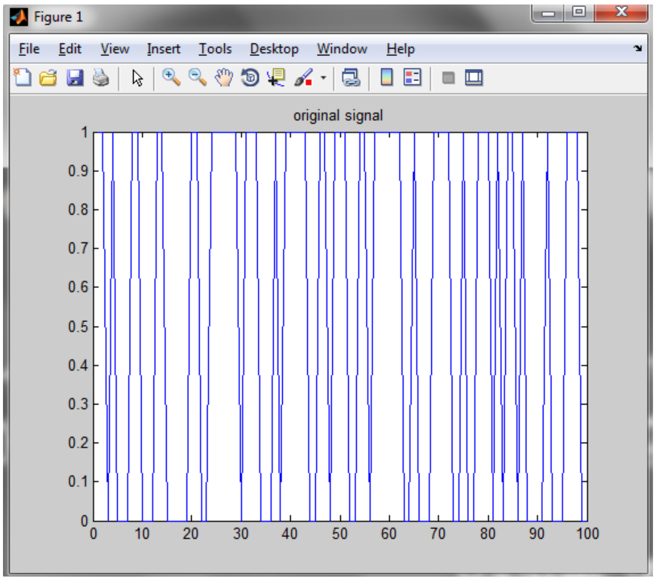

FIG: ORIGINAL INPUT SIGNAL

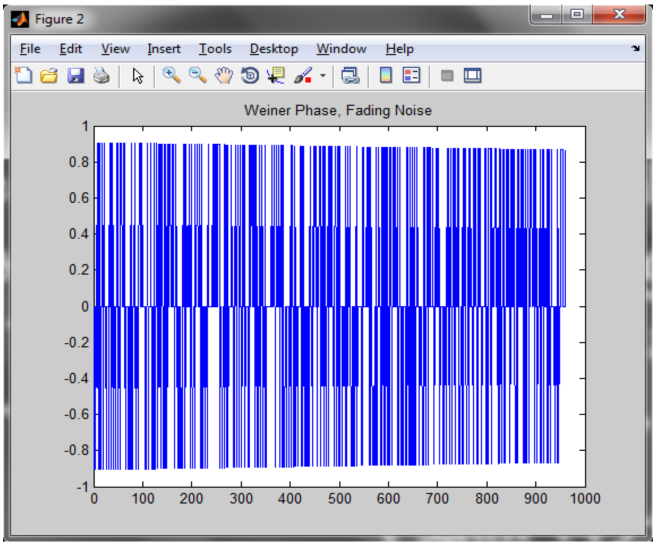

FIG: WEINER PHASE,FADING NOISE

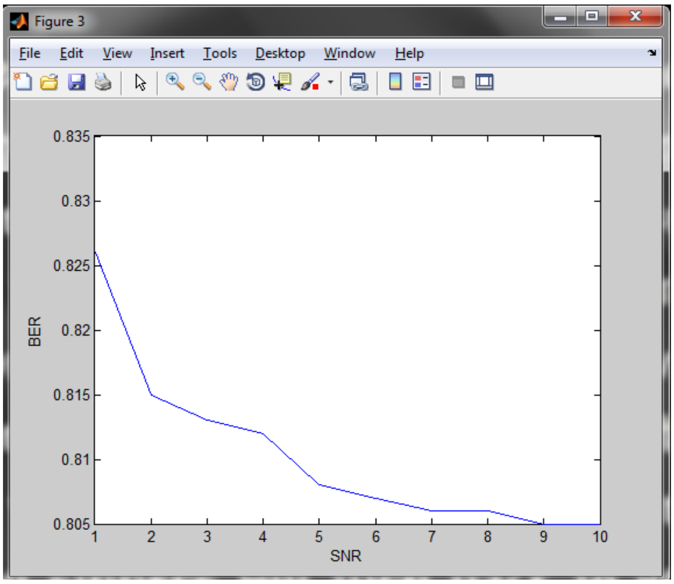

FIG: OUTPUT SIGNAL

V. CONCLUSION

The effect of Wiener phase noise impairment on the performance of OFDM systems was studied. For free-running or autonomous oscillators, the Wiener process best characterizes the phase noise. An interesting topic of future research would be to extend the analysis for phase locked loop realizations of oscillators. For the phase noise impaired OFDM system, we see that the SINR and capacity depend on two independent random variables characterizing the phase noise and the channel. The random variable characterizing Wiener phase noise was shown to be a sum of correlated gamma variables using a Taylor series approximation. The PDF for a sum of correlated gamma variables was derived and applied to the random variable describing the Wiener phase noise process. The parameters of the PDF are the eigen values which are obtained from square-root of the normalized covariance matrix of the gamma variables. Thus, any performance measure derived from the PDF depends on this ratio. It is seen from the analytical expressions, and justified by the simulations, that the performance degrades when this ratio is small.

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