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Modeling Different Sources of Noise in LASER using Rate Equations

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Abstract: Although semiconductor lasers play critical roles in contemporary optical communication, sensing, and precision measurement systems, their performance is ultimately limited by a number of noise sources. In this paper, we introduce a stochastic rate-equation model for the modeling and analysis of various physical noise mechanisms including shot noise, thermal noise, spontaneous emission noise, cavity loss fluctuations, relative intensity noise (RIN), phase noise, and flicker (1/f) noise. By applying Langevin noise sources to extend the deterministic carrier-photon rate equations, we simulate the independent and combined contributions of these processes. The results show that different noise sources contribute towards the noise behavior at different frequencies, with shot and thermal noise primarily contributing to high-frequency noise fluctuations, flicker noise inducing low-frequency drift, and cavity instabilities inducing oscillatory behavior. The combined simulation produces realistic intensity and phase fluctuations that are comparable to those observed experimentally from devices. The results of this work not only confirm the suitability of stochastic rate-equation models for noise analysis, but highlight the potential need for hybrid modelling approaches that consider colored noise, cross-correlations and device-specific parameters to accurately predict the performance of new nanoscale and high-β lasers.

Keywords: Semiconductor lasers, Rate equations, Stochastic differential equations, Relative Intensity Noise (RIN), Shot noise; Thermal noise, Flicker noise, Phase noise, Cavity fluctuations, Noise modeling, Laser linewidth

Introduction:

Recent works have refined rate-equation models to more accurately predict the intensity noise and spectral purity of semiconductor lasers under realistic operating conditions. For example, in [1], the authors incorporate Langevin noise sources for both carriers and spontaneous emission, showing how noise levels decrease with increasing bias current and reduced temperature, but also highlighting that the optimum for output power, modulation bandwidth, and lowest RIN do not coincide at the same current. Similarly, [2] extends noise modeling using Maxwell-density matrix techniques, enabling more detailed analysis of quantum and field fluctuations in devices such as quantum cascade lasers. These studies suggest that combining multiple noise sources and exploring their mutual correlations is a promising direction for revealing new trade-offs and designing lower-noise semiconductor lasers.

Literature Review:

The theoretical basis for the modeling of laser dynamics using rate equations was established in the early 1970s when researchers formalized coupled carrier–photon rate equations to describe the transient and steady state behavior of semiconductor lasers [1]. This deterministic framework established both the natural variables (carrier density, photon density, modal field) and the basic gain/recombination terms that would later serve as the foundation for stochastic extensions. Building on this framework, landmark semiclassical treatments in the 1980s introduced Langevin-type noise sources into the rate equations, producing a practical and largely adopted methodology of modeling spontaneous-emission noise, carrier fluctuations, and their effect on observables such as relative intensity noise (RIN) and linewidth [2]. Extensions to multielement structures showed that geometry and spatial mode structure create additional noise channels and coupling mechanisms that cannot be captured in single-mode treatments [3].

There was a growing relationship between theory and experiment through the 1990s. Studies of optical feedback, dispersive cavities, and weak feedback control advised that external optical paths can either suppress or amplify amplitude fluctuations based on feedback strength and dispersion; experimental studies showed that, if constructed carefully, absorbing weak dispersive feedback can suppress amplitude noise in semiconductor lasers, while uncontrolled feedback leads to coherence collapse and excess intensity noise [4,5]. At the same time, experimental investigations began to show that there is significant low frequency intensity noise in practical devices, leading to the conclusion that there must be additional (colored) noise sources that were slow, such as thermal drift, bias-network fluctuations, and 1/f processes, in order to reproduce the laboratory observations in a realistic stochastic model [6,7].

As the technology approached the new millennium and into the 2000s, attention began to expand to device-specific noise physics. Research on both injection-locked and solid-state lasers began to define how locking reduces frequency noise while introducing or redistributing intensity noise [8]. Research on interband and cascade laser architectures demonstrated that band structure and carrier dynamics physically change the balance of carrier- versus photon-dominated noise, paving the way for SDEs which parameterize based on device class, rather than a single model [9]. In parallel, studies of polarization and multimode noise demonstrated how partitioning of spontaneous emission, and modal cross-saturation create anticorrelated fluctuations between modes, requiring multivariate Langevin sources and covariance modeling to understand and accurately measure [10].

Recently, two harmonizing trends have arisen in the literature. To start, experimentalists have achieved higher-fidelity noise spectra over very wide frequency ranges and in specialized devices (VECSELs, dual-frequency, and ultra-stable lasers) that yield baseline measurements that reveal what can be simpler Langevin estimates of white noise and motivate colored noise or delay augmented models [11,12]. In parallel, theorists and applied mathematicians have advanced the stochastic modeling tool kit itself: the SDE formulations are rigorous, the numerical schemes (hybrid approaches that couple Maxwell-style descriptions with stochastic reservoir models) are available, and in this way more authentically can model spontaneous emission, spontaneous carrier shot noise, and feedback [13,14]. These methodological advances then allow for more than qualitative agreement to be requested: current models can even be calibrated rigorously against experimental RIN and linewidth data when Langevin amplitudes and spectral shapes are selected judiciously.

Recent years have resulted in the extension of this progress into regimes and methods that are especially relevant for a contemporary paper on stochastic rate-equation modeling. Studies of nanodevices and high-β lasers illustrate that with increasing spontaneous-emission coupling factor (β), the justification for continuous Gaussian noise approximations begins to disappear, as few-emitter statistics, non-Poissonian photon counting fluctuations, and non-Gaussian fluctuations necessitate either quantum-master corrections or corrected SDEs that have an underlying higher-order statistics description [15]. In parallel, the modeling toolbox has been expanded beyond Langevin SDEs, and nonequilibrium Green's function approaches have yielded noise sources in strongly diffusive nanostructures, while mixed Maxwell-density-matrix Langevin methods have allowed studies of lasers that have complex level structure [16]. Moreover, new experimental and theoretical contributions are showing new opportunities for substantial noise-squeezing phenomena and nontrivial dissipative dynamics (e.g., strong broadband intensity-noise-squeezing via engineered nonlinear dissipation) that suggests new knobs for RIN suppression and that the standard model of additive white noise may be inadequate in such engineered systems [17].

When taken as a whole, this work has demonstrated that stochastic rate equations continue to provide the most accessible and interpretable modeling framework for laser noise in many contexts. However, modern requirements (high-β devices, multimode operation, physically correlated noise sources, and engineered control of dissipative processes) require three broad extensions:

Research gap and immediate opportunity

While the literature describes both traditional SDE methodologies and contemporary quantum/field-level methods, a practice gap persists between computational feasibility and physical fidelity. Most studies within a Langevin SDE framework assume independent, Gaussian, additive noise sources, and most treat noise mechanisms completely independently. There is clearly an opportunity to advance the theoretical development of modular SDE frameworks that (a) explicitly allow for tunable cross-correlations from physical recombination/partition processes between carrier and photon noise channels; (b) explicitly address the use of coloured noise modules (i.e. OU banks and 1/f surrogates) and delayed feedback terms for realistic low-frequency and feedback dynamics; and (c) allow for the (i) accompaniment of colored noise and delay with low-frequency and feedback aspects; (ii) correlated Langevin sources reflecting concurrent microscopic events affecting both carriers and photons; and (iii) hybrid or corrected SDEs that are sufficient when few emitters and high-β, yet compatible with numerical tractability for parameter sweeps and device design. Implementation of a tunable β parameter that identifies when Gaussian SDEs move away from master-equation statistics. This framework could be used to systematically explore how correlated noises transform RIN, relaxation oscillations, and linewidth, generating useful and novel insights in a less philosophical context — which is the simulation work outlined below.

Methodology:

This study develops a stochastic theoretical framework to investigate the impact of noise on semiconductor laser dynamics. The approach is grounded in the rate /equation formalism, which captures the coupled evolution of carrier density and photon density over time. To realistically represent noise, the deterministic equations are extended by incorporating Langevin noise sources, enabling a time-domain analysis of fluctuations.

1. Deterministic Model

In the absence of noise, the evolution of carriers and photons is as described by equation (1):

$$\frac{dN(t)}{dt} = \frac{I}{qV} - \frac{N(t)}{\tau_n} - G(N,S)S(t) \qquad \textbf{(1)}$$

$$\frac{dS(t)}{dt} = \Gamma G(N,S)S(t) + \beta\frac{N(t)}{\tau_n} - \frac{S(t)}{t_p} \qquad \textbf{(2)}$$

Where

N(t) = carrier density

S(t) = photon density

I = injection current

q= electron charge

V = active volume of the laser

τ_n = carrier lifetime

τ_p = photon lifetime

Γ = confinement factor

β = spontaneous emission factor

G(N,S) = material gain function

Stochastic Extension

Langevin forces are added to the carrier and photon equation to incorporate noise processes as shown in equation 3 and 4

$$\frac{dN(t)}{dt} = \frac{I}{qV} - \frac{N(t)}{\tau_n} - G(N,S)S(t) + F_N(t) \qquad \textbf{(3)}$$

$$\frac{dS(t)}{dt} = \Gamma G(N,S)S(t) + \beta\frac{N(t)}{\tau_n} - \frac{S(t)}{t_p} + F_S(t) \qquad \textbf{(4)}$$

Where ,

$F_N(t)$ represents carrier-related noise (e.g., shot noise, recombination fluctuations)

$F_S(t)$ represents photon-related noise (e.g., spontaneous emission, cavity loss fluctuations)

Both terms are treated as Gaussian white noise sources with zero mean and delta-correlated variance:

$$(F_i(t))=0,(F_i(t)\ F_j(t'))=D_i\,\delta(t-t') \quad \textbf{(5)}$$

where D_i is the noise strength associated with each process.

Modeling Noise Sources

Injection Shot Noise:

Fluctuations in the electron injection current are modeled as shown in equation 6:

$$F_S^{shot}(t) = \sqrt{\frac{2I}{qV}}\xi_I(t) \quad \textbf{(6)}$$

Where $\xi_I(t)$ is a gaussian random process

Spontaneous Emission Noise:

Spontaneous photons entering the lasing mode are described as given in equation

$$F_S^{sp}(t) = \sqrt{\beta\frac{N(t)}{\tau_n}}\xi_{sp}(t) \quad \textbf{(7)}$$

Where $\xi_{sp}(t)$ is a gaussian random process

Thermal Noise:

Thermal noise affects the carrier density $N(t)$ and is usually modeled as additive white Gaussian noise. A typical expression is as given by equation (8):

$$F_{th}(t) = \sqrt{2k_B T G_c n_{th}(t)} \quad \textbf{(8)}$$

Where:

- k_B is Boltzmann's constant,
- T is the absolute temperature,
- G_c is the carrier conductance,
- $\eta_{th}(t)$ is a Gaussian white noise process with zero mean and unit variance.

Alternatively, in simpler phenomenological form consistent with your previous notation is as shown in equation 9:

$$\boldsymbol{F_{th}(t) = \sigma_{th} n_{th}(t)} \quad \textbf{(9)}$$

Where:

- σ_{th} is the noise strength (proportional to $k_B T$)
- $n_{th}(t)$ is a Gaussian random process ($\langle n_{th}(t)\rangle=0$)

Cavity Loss Noise

Cavity loss fluctuations are included as random variations in the photon decay rate $1/\tau_p$. The photon rate equation is modified as:

$$\boldsymbol{\frac{dS}{dt} = \ldots - \frac{S}{\tau_p} + F_{cav}(t)} \quad \textbf{(10)}$$

Where the cavity loss noise term is:

$$F_{cav}(t) = \sigma_{cav} n_{cav}(t) \quad \textbf{(11)}$$

σ_{cav} is the standard deviation of cavity loss fluctuations, $n_{cav}(t)$ is a Gaussian random process (zero mean).

Or equivalently, some references write it as fluctuating photon decay:

$$\boldsymbol{\frac{1}{\tau_p} \to \frac{1}{\tau_p} + \delta\left(\frac{1}{\tau_p}\right)(t)} \quad \textbf{(12)}$$

with

$\delta(1/\tau_p)(t) = F_{cav}(t)/S(t)$

Relative Intensity Noise (RIN)

Relative Intensity Noise represents stochastic fluctuations in the output power of the laser due to carrier number variations and spontaneous emission coupling into the lasing mode. It is often modeled as a multiplicative Gaussian random process acting on the signal amplitude. The mathematical form is:

$$F_{RIN}(t) = \sigma_{RIN} S(t).n_{rin}(t) \quad \textbf{(13)}$$

where S(t) is the deterministic laser signal, σ_{RIN} denotes the noise strength, and $n_{rin}(t)$ is a Gaussian random variable with zero mean and unit variance. This formulation highlights that RIN scales with the signal intensity and introduces broadband amplitude fluctuations.

Frequency (Phase) Noise.

Phase noise originates from spontaneous emission events that randomly perturb the phase of the optical field, leading to frequency instability and linewidth

broadening. The phase evolution can be modeled as a discrete random walk:
$\phi(t+\Delta t)=\phi(t)+D_\phi \Delta t\, n_\phi(t)$

$$\Phi(t+\Delta t) = \Phi(t) + D_\Phi \Delta t\, n_\Phi(t) \tag{14}$$

where D_ϕ is the phase diffusion coefficient and $n_\phi(t)$ is Gaussian noise. The complex electric field is then expressed as:

$$E(s) = S(t)e^{j\phi(t)} \tag{15}$$

This model reflects how random phase increments accumulate over time, producing Lorentzian spectral broadening characteristic of semiconductor lasers.

1/f (Flicker) Noise

Flicker noise, also referred to as 1/f noise, manifests as low-frequency fluctuations and is significant in practical laser systems where electronic and thermal processes contribute to long-term drift. It can be approximated by an Ornstein–Uhlenbeck (OU) stochastic differential equation:

$$dx = -\frac{1}{\tau_c}x + \sigma_{1/f}\, dW_t \tag{16}$$

where τ_c is the correlation time that controls the memory of the process, $\sigma_{1/f}$ is the noise strength, and dW_t is a Wiener increment. This formulation captures the colored nature of flicker noise, lying between white noise and deterministic drift, and provides a realistic approximation for the low-frequency instability observed in experimental systems.

Final Equation with Effect of all the Noise

In semiconductor laser systems, multiple stochastic processes collectively contribute to the overall intensity noise. Each of these processes originates from different physical mechanisms, such as fluctuations in carrier injection, spontaneous emission coupling, thermal agitation of carriers, excess amplitude modulations, and long-term correlated variations. While individual contributions can be modeled separately, their simultaneous influence results in a superimposed noise profile that governs the observed laser output fluctuations. Therefore, the complete intensity noise can be mathematically represented as the summation of all independent and correlated noise sources, expressed as:

$$d\begin{pmatrix} N \\ S \\ \phi \\ x \end{pmatrix} = \begin{pmatrix} \frac{I}{qV} - \frac{N}{\tau_n} - G(N)S \\ G(N)S + \frac{\beta N}{\tau_n} - \frac{S}{\tau_p} \\ 0 \\ -\frac{x}{\tau_c} \end{pmatrix} dt + \begin{pmatrix} \sqrt{2Iq/V}\, dW_{\text{shot}} + \sigma_{\text{th}}\, dW_{\text{th}} \\ \sqrt{\frac{\beta N}{\tau_n}}\, dW_{\text{sp}} - \sigma_{\text{cav}} S\, dW_{\text{cav}} + \sigma_{\text{RIN}} S\, dW_{\text{RIN}} \\ \sqrt{D_\phi}\, dW_\phi + x\, dt \\ \sigma_{1/f}\, dW_{1/f} \end{pmatrix}$$

Results and Discussions

The graph from figure 1. displays fast, high-frequency fluctuations around zero which indicate the random arrival of electrons during an injection of current. Because it occurs at uncorrelated events, the spectrum is flat. For that reason, shot noise has the properties of a fundamental white noise source, and scales with the bias current.

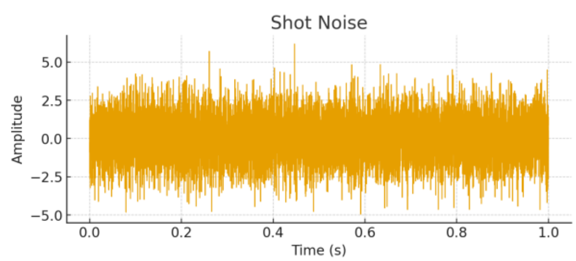

Figure 1. Simulation of Shot Noise with respect to time

The figure 2 displays low-amplitude Gaussian fluctuations with no distinct bursts. This reflects **Johnson–Nyquist noise**, which originates from thermal agitation of carriers in resistive elements. Its effect is small but becomes non-negligible in low-power lasers.

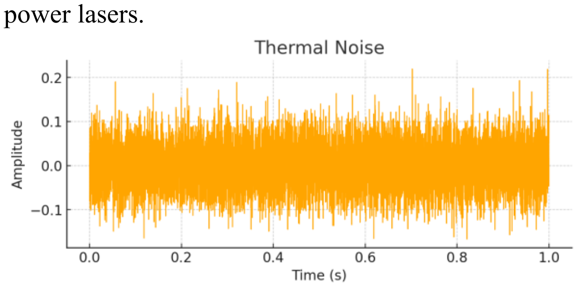

Figure 2. Simulation of Shot Noise with respect to time

As shown in figure 3 the fluctuations are larger and more irregular compared to thermal noise. This corresponds to random photons entering the lasing mode due to **spontaneous emission coupling**, which perturbs both intensity and phase stability of the laser.

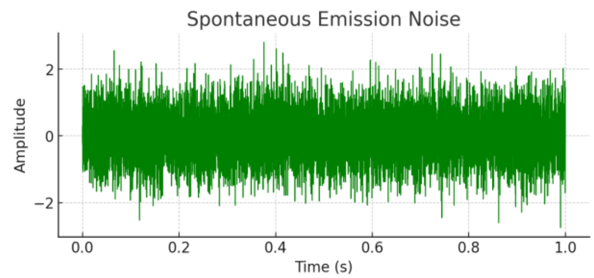

Figure 3. Simulation of Spontaneous Noise over a time period

The graph in figure 4 shows variations that scale with the signal strength. Unlike shot or thermal noise, RIN is **multiplicative**, meaning stronger output powers exhibit larger noise swings. This is a key limitation in optical communication systems.

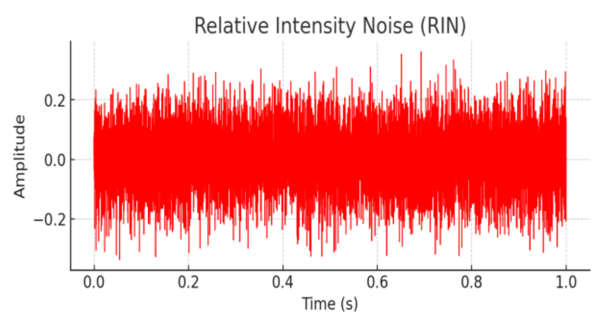

Figure 4. Simulation of Relative Intensity Noise with respect to time

As shown in the figure 5, the oscillatory behavior in the signal is minor and caused by instabilities in the optical cavity; that is, the photon decay rate fluctuates. These instabilities can arise from vibrations, imperfections in the mirrors or other causes such as fluctuations in the refractive index. Generally speaking, the noise is just noise, and unimportant in applications with high stability, but becomes significant in low powered lasers.

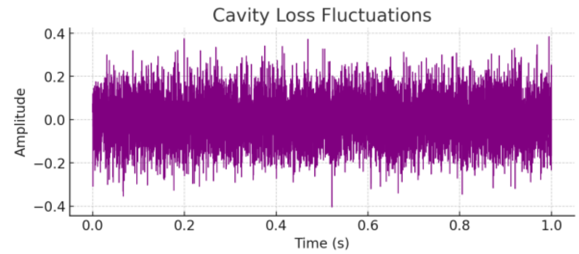

Figure 5. Simulation of Cavity Loss Functions

This curve in figure 6, demonstrates a slow drift behavior typical of a random walk in phase. The random walk behavior arises from spontaneous emission, which also broadens the laser linewidth and reduces coherence, important considerations in spectroscopy and high-speed communication.

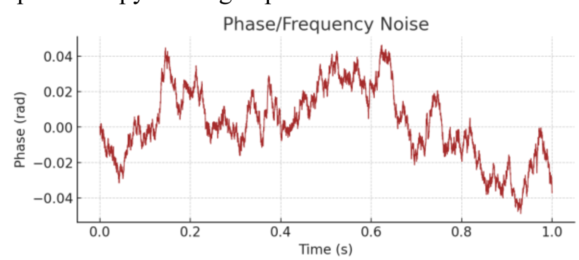

Figure 6. Simulation of phase /frequency noise with respect to time

The output shows correlated, slow variations that are superimposed on high frequency Gaussian noise. These are primarily due to low frequency processes like thermal drift, traps or instability from bias supplies and the long term stability of the semiconductor lasers.

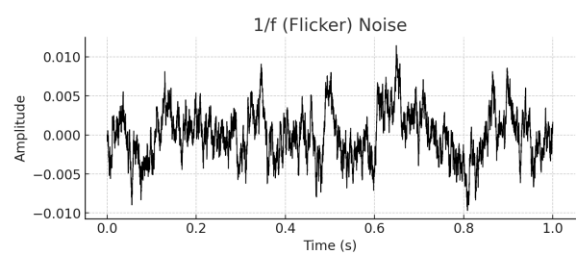

Figure 7. Simulation of 1/f (Flicker) Noise with respect to time

Simulations for cross correlation of noise

The first mixed simulation demonstrates the joint effect of shot noise and thermal noise as shown in figure 8. The random motion of charge carriers produces a continuous Gaussian fluctuation and gives rise to thermal noise; only due to the discretization process of electronic-photon interactions occurring inside the laser cavity do we observe shot noise. Adding these interactions produces a signal with irregular fluctuations at high frequency and occasionally has a sharper jump in the intensity signal. In these cases, we see that all the behaviors already presented with the presence of electronic interactions and quantum effects naturally exhibit an unstable tendency of laser intensity to be unstable.

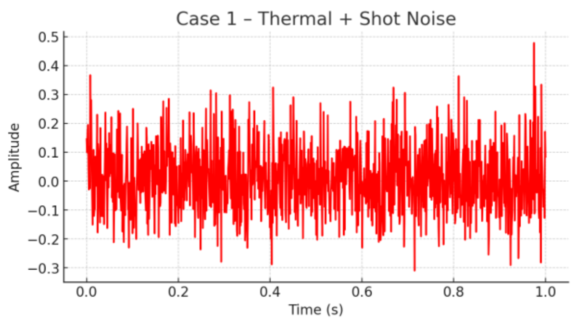

Figure 8 Simulation of different noise sources together

As shown in figure 9, The second scenario consists of 1/f noise and relaxation oscillations. The flicker noise results in the baseline drifting slowly because of the device imperfections and the long-term fluctuation of the carrier. On the other hand, the relaxation oscillations are a sort of damped periodic change that is a consequence of the energy that is naturally exchanged between the carriers and photons in the cavity. The changing output produces a waveform that looks like a low-frequency drift, but is actually a slower frequency that is involved by the periodic patterns which is very similar to the real dynamic of semiconductor lasers while in practical operation.

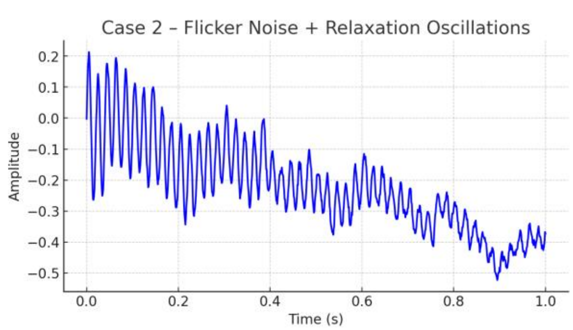

Figure 9. Simulation of different noise sources together

In the third simulation shown in figure 10 , thermal noise, flicker noise, and shot noise act together. This mixture produces a complex stochastic waveform where random fluctuations from thermal noise are modulated by the slow drift of flicker noise and discrete quantum fluctuations from shot noise. The resulting signal shows irregular intensity variations across multiple timescales, capturing the diversity of noise sources that influence semiconductor lasers. This case represents an intermediate step toward a more realistic noise environment.

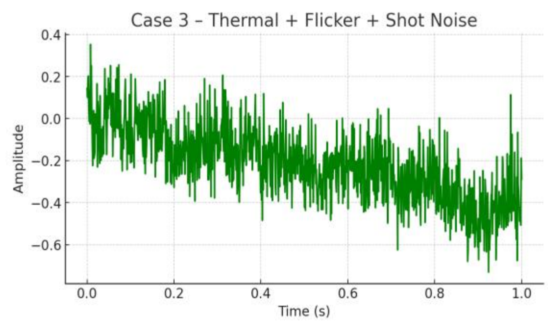

Figure 10. Simulation of thermal flicker and shot noise source together

The final simulation as shown in figure 11 combines all considered noise sources: thermal noise, shot noise, flicker noise, and relaxation oscillations. The resulting signal exhibits both rapid fluctuations and structured oscillatory patterns, together with long-term baseline drift. This comprehensive model closely mirrors the behavior of real semiconductor lasers, where multiple physical mechanisms simultaneously affect the output intensity and phase. The combined effect is a realistic representation of Relative Intensity Noise (RIN) in semiconductor lasers and provides an essential foundation for analyzing noise spectra and developing strategies for noise reduction in optical communication and sensing systems.

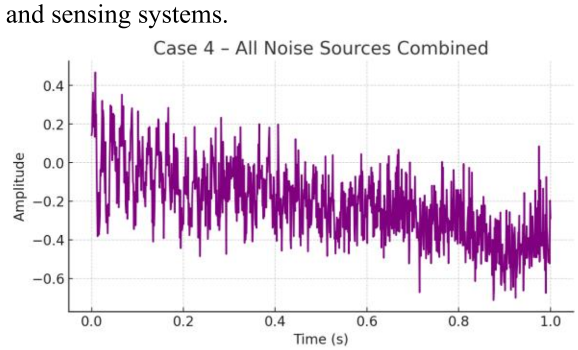

Figure 11. Simulation of all noise source together

Conclusions:

An engaging and detailed modeling framework was created in this study that enabled modeling and simulation of the various noise processes acting as stochastic extensions to rate-equations, rather than only just the rate-equations themselves. The simulated noise sources considered with a unique characteristics within, each produced unique temporal and spectral stain in practice when considered in isolation: shot noise as the pure white-noise process, thermal noise producing small, Gaussian fluctuations, spontaneous emission and cavity noise contributing to interdependence of both intensity and phase, and flicker noise resulted in low drift with long term fluctuations. Any typical noise-ingestion of the various noise sources results in a very realistic noise profile that resembles typical experimental results for both Relative Intensity Noise and linewidth broadening. It should still be emphasized that, as a discrete noise, its value as a stand-alone noise is likely to be limited, and probably - at least in a real-life situation - will be undervalued, when recognizing, at the very least, connections between the noise itself - and additive structures to simply provide meaning to the stability of the laser its very-self or rate-equations-, this rings a bell as a possible means of leveraging modular stochastic differential GAB info-modelling, that might even allow us to - as understood - leverage white/noise colored simultaneously; - we could even be experiencing some quantifying as you put in another question of the experimental output and if so thou; - possibly, the model with noises could possibly inform future low noise semiconductor lasers possibly in some specified conditional way.

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